Solution to "Paper" Homework #4

Section 2.7 - Prob 51 Let us use the definition of derivative:

$$\begin{aligned} f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} \\ &= \lim_{h\to 0}\frac{|x+h-6|-|x-6|}{h} \\ f'(6) &= \lim_{h\to 0}\frac{|6+h-6|-|6-6|}{h} \\ &= \lim_{h\to 0}\frac{|h|}{h} \end{aligned}$$

Use the definition of absolute function:

$$|x| = \begin{cases} x & \text{if} \quad x \geq 0 \\ -x & \text{if} \quad x < 0 \end{cases}$$

So:

$$\frac{|h|}{h} = \begin{cases} \frac{h}{h} = 1 & \text{if} \quad h > 0 \\ \frac{-h}{h} = -1 & \text{if} \quad h < 0 \end{cases}$$

Hence:

$$\lim_{h\to 0^-}\frac{|h|}{h} = -1 \quad \text{and} \quad \lim_{h\to 0^+}\frac{|h|}{h} = 1$$

So: $\lim_{h\to 0}\dfrac{|h|}{h}$ does not exist; thus $f(x)$ is not differentiable at 6.

Section 3.1 - Prob 2

(a)

(b) $f(x) = e^x$ is an exponential function. $g(x) = x^e$ is a power function. We have:

$$f'(x) = e^x \quad \text{but} \quad g'(x) = e\,x^{e-1}$$

(c) $f(x)$ grows faster when x is large.

Section 3.2 - Prob 51 Compute the derivative $f'(x) = e^x(x^3+3x^2) = e^x x^2(x+3)$. $f(x)$ is increasing if $f'(x) > 0$, and since $e^x > 0$ and $x^2 \geq 0$, we can conclude that $f(x)$ is increasing when $x+3 > 0$ and $x \neq 0$. So the interval we are looking for is $I = (-3,0)\cup(0,+\infty)$.

Section 3.2 - Prob 52 Compute the second derivative: $f''(x) = (x^2+4x+2)e^x$. $f(x)$ concaves downward when $f''(x) < 0$ which happens when $x^2+4x+2 < 0$ (since $e^x > 0$). $x^2+4x+2 = 0$ has two roots: $x_1 = -2-\sqrt{2}$ and $x_2 = -2+\sqrt{2}$; so, by using a sign test or just looking at the graph of the parabolic function x^2+4x+2, we conclude that $x^2+4x+2 < 0$ when $-2-\sqrt{2} < x < -2+\sqrt{2}$.

Section 3.3 - Prob 34 Compute $f''(x) = -2(\sec x)^2\tan(x)$. f concaves downward when $f'' < 0$, so $\tan(x) > 0$ which means $0 \leq x < \frac{\pi}{2}$.