KU THE UNIVERSITY OF KANSAS

Department of Mathematics

Math 115 – Spring 2013

The final exam is on
WEDNESDAY, MAY 15, 4:30pm –7:00pm.
Room: Budig 120.

The final exam will covered the following sections from the textbook.

- Chapter 2, Sections 2.2, 2.3, 2.4, 2.5, 2.6.
- Chapter 3, Sections 3.1, 3.2, 3.3, 3.4, 3.5, 3.6, 3.7.
- Chapter 4, Sections 4.1, 4.2, 4.3, 4.4, 4.5
- Chapter 5, Sections 5.1, 5.2, 5.4, 5.5
- Chapter 6, Sections 6.1, 6.3, 6.4, 6.5

These problems are intended to be used as part of a review for the final exam. There is also an assignment of review problems posted in WebAssign.They are not a substitute for studying the material of the sections and all the homework assignments.

MULTIPLE CHOICE QUESTIONS: CIRCLE THE MOST CORRECT ANSWER. **No partial credit.**

1. Evaluate: $\lim_{x\to 3} \dfrac{x^2-9}{x+3}$.

(A) 6 (B) 0 (C) 9 (D) Does not exist.

(E) None of the above.

2. Evaluate: $\lim_{x\to \infty} \dfrac{3x^3-x+9}{7x^3-1}$.

(A) $\dfrac{7}{3}$ (B) 7 (C) $\dfrac{3}{7}$ (D) Does not exist.

(E) None of the above.

3. The equation of the tangent line to the curve $y = \dfrac{x}{x+1}$ at $(1, \frac{1}{2})$ is

$(A)\ y = \dfrac{1}{4}(x-1) - \dfrac{1}{2}$ $(B)\ y = \dfrac{1}{2}(x-1) - \dfrac{1}{4}$ $(C)\ y = \dfrac{1}{2}(x-1) + \dfrac{1}{4}$

$(D)\quad y = \dfrac{1}{4}(x-1) + \dfrac{1}{2}$

(E) None of the above.

Use the graph of the function f below to answer Problems 3, 4, 5, and 6

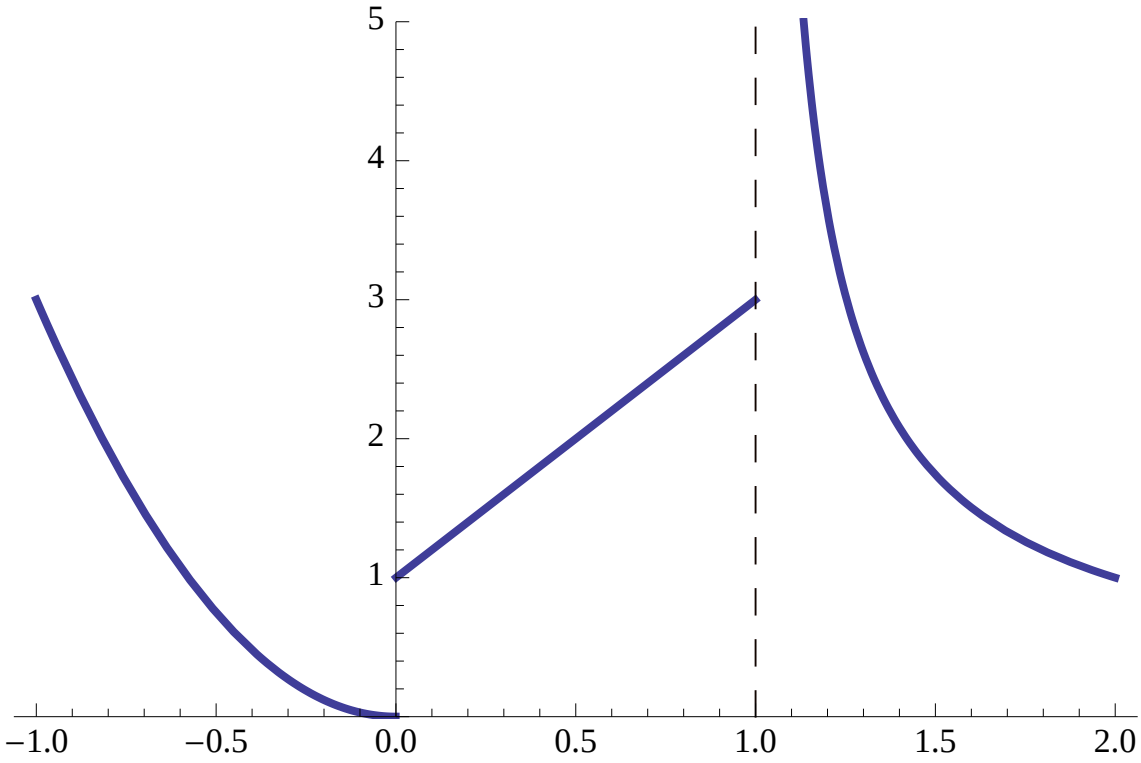

FIGURE 1

4. $\lim\limits_{x \to 1^+} f(x) =$

$(A)\ 0$ $(B)\ 3$ $(C)\ 11$ (D) Does not exist.

(E) None of the above.

5. $\lim\limits_{x \to 0^-} f(x) =$

$(A)\ 1$ $(B)\ 0$ $(C)\ 2$ (D) Does not exist.

(E) None of the above.

6. The function f is continuous at

$(A)\ x = -0.5, 0, \text{and } 1.5 \quad (B)\ x = -0.5, 1, \text{and } 1.5 \quad (C)\ x = -0.5, 0, \text{and } 0.5 \quad (D)\ x = -0.5, 0.5, \text{and } 1.5$

(E) None of the above.

7. The function f is not continuous at

$(A)\ x = -0.5 \qquad (B)\ x = 0.5 \qquad (C)\ x = 1 \qquad (D)\ x = 1.5$

(E) None of the above.

8. The equation of the tangent line to the curve $y = \mathrm{e}^{\sqrt{x+1}}$ at $(0, \mathrm{e})$ is

$(A)\ y = 2x + 1 \qquad (B)\ y = \mathrm{e}\,x + \dfrac{\mathrm{e}}{2} \qquad (C)\ y = \dfrac{x}{2} + 1 \qquad (D)\ y = \dfrac{\mathrm{e}}{2}x + \mathrm{e}$

(E) None of the above.

9. Given the equation $x^2 - 2x^2y + y^2 = 1$. Find the derivative $\dfrac{dy}{dx}$ by implicit differentiation.

$(A)\ \dfrac{x - 2xy}{2y - x} \qquad (B)\ \dfrac{2y - x}{x - 2xy} \qquad (C)\ \dfrac{2y - x}{x - y} \qquad (D)\ -\dfrac{x - y}{2y - x}$

(E) None of the above.

Answer Problems 10, 11, and 12 for the following information.

A manufacter has a monthly fixed cost of $\$65,000$ and a production cost of $\$23$ for each unit produced. The produce sells for $\$30$ per unit.

10. Find the cost function.

$(A)\ C(x) = 65,000 - 23x \qquad (B)\ C(x) = 65,000 + 23x$

$(C)\ C(x) = 65,000 - 30x \qquad (D)\ C(x) = 65,000 + 3x$

(E) None of the above.

11. Find the profit function P.

$(A)\ P(x) = 65,000 - 23x \qquad (B)\ P(x) = 65,000 + 23x$

$(C)\ P(x) = 65,000 - 30x \qquad (D)\ P(x) = 65,000 + 3x$

(E) None of the above.

12. Find the marginal profit function.

$(A)\ 23 \qquad (B)\ 23x \qquad (C)\ 3 \qquad (D)\ 3x$

(E) None of the above.

13. The function $f(x) = x^3 - 3x$ is increasing for

(A) all real numbers $\quad (B)\ x$ in $(-1, 1) \quad (C)\ x$ in $(-\infty, 1) \quad (D)\ x$ in $(-\infty, 1)$ and x in $(1, \infty)$

(E) None of the above.

14. The function $f(x) = \dfrac{6x^2}{x^2 + 25}$ is decreasing for

(A) all real numbers $\quad (B)\ x$ in $(0, \infty) \quad (C)\ x$ in $(-\infty, 0) \quad (D)\ x$ in $(-\infty, 0)$ and x in $(0, \infty)$

(E) None of the above.

Answer Problems 15, 16, and 17 for the following information.

Let $f(x) = x^4 - 18x^2 + 5$.

15. The function $f(x)$ is concave upward on the interval :

$(A)\ (-\infty, -2) \qquad (B)\ (2, \infty) \qquad (C)\ (-\infty, -\sqrt{3})$ and the interval $(\sqrt{3}, \infty) \qquad (D)\ (-\sqrt{3}, \sqrt{3})$

(E) None of the above.

16. The function $f(x)$ is concave downward on the interval:

(A) $(-\infty, -2)$ (B) $(2, \infty)$ (C) $(-\infty, -\sqrt{3})$ and the interval $(\sqrt{3}, \infty)$ (D) $(-\sqrt{3}, \sqrt{3})$

(E) None of the above.

17. The inflection point(s) of $f(x)$ is (are):

(A) $(-3, -76)$ and $(3, -76)$ (B) $(0, 0)$ (C) $(-\sqrt{3}, -40)$ and $(\sqrt{3}, -40)$

(D) $(-4, 0)$ and $(4, 0)$ (E) No inflection points.

18. How many critical points does $\dfrac{8x}{x^2+7}$ have?

(A) None. (B) 1 (C) 2 (D) 3

(E) None of the above.

19. Consider $y = x^4 - 1$. The point $(0, -1)$ is:

(A) a relative minimum. (B) a relative maximum.

(C) an inflection point. (D) an absolute minimum.

(E)None of the above.

20. A firm has determined that its revenue $R(x)$ (measured in thousands of dollars) from the sale of x units of its product is $R(x) = 80 - \dfrac{200}{x+4} - 2x$. The value of x that maximizes revenue is

(A) 30 *units*. (B) 8 *units*. (C) 6 *units*. (D) 10 *units*. (E) 0 *units*.

21. An efficiency study conducted for a packing company showed that the number of cases packed by the average worker t hours after starting work at 8a.m. $(t = 0)$ is given by $E(t) = -t^3 + 4t^2 + 12t, \ \ 0 \leq t \leq 4$. At what time during the the morning shift is the average worker at peak efficiency?

$$(A)\ 10:30\ a.m. \qquad (B)\ 9:20\ a.m. \qquad (C)\ 11:00\ a.m.$$
$$(D)\ 10:00\ a.m. \qquad (E)\ 10:15\ a.m.$$

22. Which of the following statements is true for the function represented in the graph below in Figure 2?

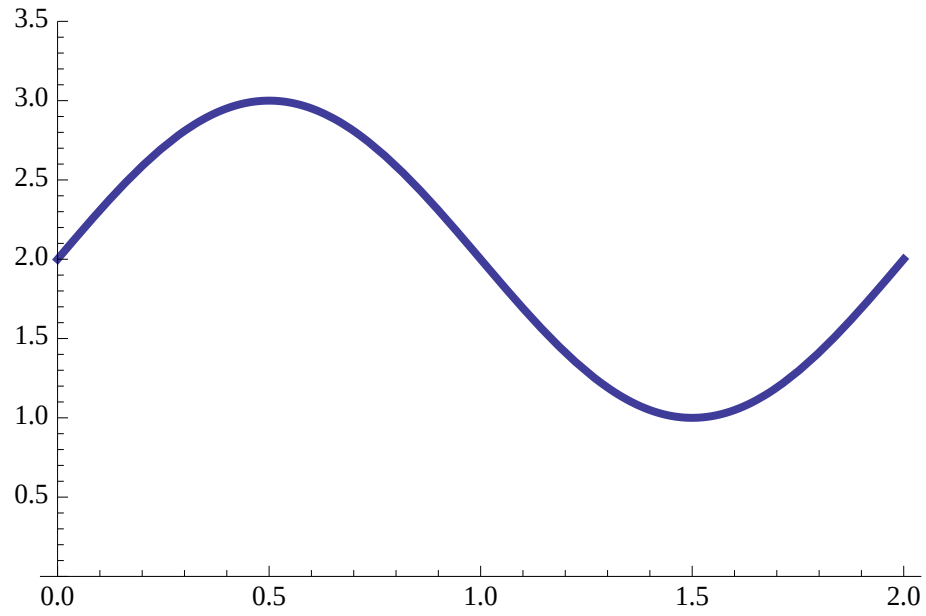

FIGURE 2

(A) $f' > 0$ on $(0, 0.5)$, $f'' > 0$ on $(0, 1)$ and $f'' < 0$ on $(1, 2)$.
(B) $f' > 0$ on $(0, 0.5)$, $f'' < 0$ on $(0, 1)$ and $f'' > 0$ on $(1, 2)$.
(C) $f' < 0$ on $(0, 0.5)$, $f'' > 0$ on $(0, 1)$ and $f'' < 0$ on $(1, 2)$.
(D) $f' < 0$ on $(0, 0.5)$, $f'' < 0$ on $(0, 1)$ and $f'' > 0$ on $(1, 2)$.
(E) None of the above

23. Which of the following statements is true for the function represented in the graph below in Figure 3?

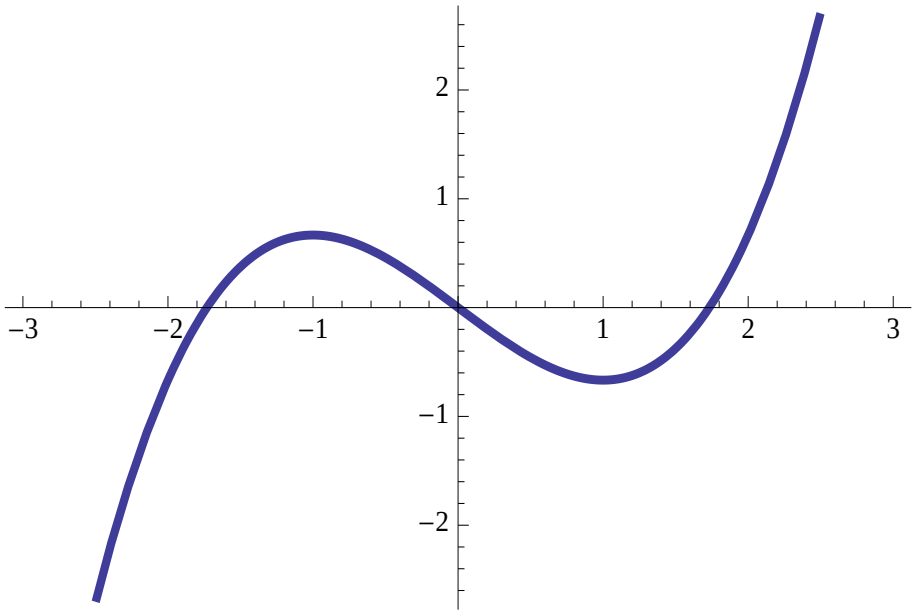

FIGURE 3

(A) f has one inflection point and one local minimum in the interval $(-3, 3)$.

(B) f has one inflection point, one local maximum and one local minimum in the interval $(-3, 3)$.

(C) f has two inflection points and one local minimum in the interval $(-3, 3)$.

(D) f has two inflection points and one local maximum in the interval $(-3, 3)$.

(E) None of the above

24. Find the vertical asymptote(s) for the graph of $f(x) = \dfrac{3x^2 - 17x - 1}{x^2 - x - 22}$.

$(A)\ x = -1$ and $x = 2$ $(B)\ y = 3$ $(C)\ y = -1$ and $y = 2$ $(D)\ x = 1$ and $x = -2$

(E) None of the above.

25. Find the horizontal asymptote(s) for the graph of $f(x) = \dfrac{3x^2 - 17x - 1}{x^2 - x - 22}$.

$(A)\ x = -1$ and $x = 2$ $(B)\ y = 3$ $(C)\ y = -1$ and $y = 2$ $(D)\ x = 1$ and $x = -2$

(E) None of the above.

26. The derivative of the function $\ln(\mathrm{e}^x + x)$ is

$(A)\ \dfrac{\mathrm{e}^x + 1}{\mathrm{e}^x + x}$ $(B)\ 1$ $(C)\ (\mathrm{e}^x + 1)\ln(\mathrm{e}^x + x)$ $(D)\ (\mathrm{e}^x + x)\ln(\mathrm{e}^x + x)$ $(E)\ \dfrac{\mathrm{e}^x + x}{\mathrm{e}^x + 1}$

27. The derivative of the function $\mathrm{e}^{\sqrt{x+1}}$ is

$(A)\ \mathrm{e}^{\sqrt{x+1}}$ $(B)\ \dfrac{\mathrm{e}^{\sqrt{x+1}}}{\sqrt{x+1}}.$ $(C)\ \dfrac{\mathrm{e}^{\sqrt{x+1}}}{2\sqrt{x+1}}$

$(D)\ \dfrac{\mathrm{e}^{\sqrt{x+1}}}{2(x+1)}.$ $(E)\ 2\sqrt{x+1}\,\mathrm{e}^{\sqrt{x+1}}.$

Answer Problems 28, 29, and 30 for the following information.

A secretary's typing speed (in words per minute) after t years of experience can be modeled by the equation $S(t) = 90 - 32\mathrm{e}^{-0.69t}$.

28. Find the typing speed at one year of experience. (Up to two decimal places.)

$(A)\ 58$ $(B)\ 106$ $(C)\ 70$ $(D)\ 26$ $(E)\ 73.95$

29. What is instantaneous rate of change of the typing speed at one year of experience?.(Up to two decimal places.)

$(A)\ 0.35$ $(B)\ 11.07$ $(C)\ 16.1$ $(D)\ -16.1;$ $(E)\ -11.07$

30. How long will it take to reach a typing speed of 80 words per minute? Give your answer rounded to nearest year.

$(A)\ 1\,year.$ $(B)\ 2\,years.$ $(C)\ 3\,years.$ $(D)\ 4\,years.$ $(E)\ 5\,years.$

Answer Problems 31 and 32 for the following information.

A company has determined that the value V (in dollars) of its investments is

$$V(t) = 20,000e^{0.02t},$$

where t is the number of years since the investments were made.

31. Find the value of the company's investments after five years.(Rounded up to dollars)

$(A)\ \$442.$ $(B)\ \$54,366.$ $(C)\ \$2,210.$
$(D)\ \$22,103.$ $(E)\ \$5,437.$

32. The instantaneous rate of change of the value of the investements after 5 years since the investments were made is (rounded up to dollars per year).

$(A)\ 442\,dollars/year.$ $(B)\ 54,366\,dollars/year.$ $(C)\ 2,210\,dollars/year.$
$(D)\ 22,103\,dollars/year.$ $(E)\ 5,437\,dollars/year.$

Answer Problems 33, 34, and 35 for the following information.

In an experiment involving an unidentified virus, researchers have determined that the body temperature (B) of a victim varies with respect to time (t) and is approximated by

$$B(t) = \frac{10t}{t^2+1} + 98.6,$$

where B is measured in degrees Fahrenheit and t is measured in hours.

33. What is the value of the body temperature after 2 hours? (Rounded up to two decimal places.)

$(A)\ 98.6°F$ $(B)\ 103.6°F$ $(C)\ 102.6°F$ $(D)\ 103.3°F$ $(E)\ 102.1°F$

34. Determine the rate of change of body temperature with respect to time.

$(A)\ -10\dfrac{t^2+1}{(t^2-1)^2}$ $(B)\ -10\dfrac{2t}{(t^2+1)^2}$ $(C)\ -10\dfrac{t^2-1}{t^2+1}$
$(D)\ \dfrac{t^2-1}{(t^2+1)^2}$ $(E)\ -10\dfrac{t^2-1}{(t^2+1)^2}$

35. What is the average rate of change of the body temperature during the three hours after the victim was affected by the virus ?(Rounded up to two decimal place.)

$(A)\ 1\,degrees/hour \qquad (B)\ 2\,degrees/hour \qquad (C)\ 0.88\,degrees/hour$

$(D)\ 1.01\,degrees/hour \qquad (E)\ 2.01\,degrees/hour$

36. The area of the region under the graph of the function $f(x) = x^3 + x$ on the interval $[0, 1]$ is

$(A)\ 2 \qquad (B)\ \dfrac{7}{12} \qquad (C)\ 0 \qquad (D)\ 4 \qquad (E)\ \dfrac{3}{4}$

37. The area of the region under the graph of the function $f(x) = \mathrm{e}^x$ on the interval $[0, 1]$ is

$(A)\ \mathrm{e} \qquad (B)\ \mathrm{e}^2 - \mathrm{e} \qquad (C)\ 0 \qquad (D)\ -1 \qquad (E)\ \mathrm{e} - 1$

38. The indefinite integral $\int \left(\dfrac{1}{x} + 2x\right) dx$ is

$(A)\ \ln|x| + 2x^2 + C. \qquad (B)\ \ln|x| + \dfrac{1}{2}x^2 + C. \qquad (C)\ \mathrm{e}^x + x^2 + C.$

$(D)\ \ln|x| + x^2 + C. \qquad (E)\ \mathrm{e}^x + x^3 + C.$

39. The function $f(x)$ which satisfies the initial value problem $f'(x) = x^3 + x^2;\ \ f(0) = 3$ is

$(A)\ \dfrac{x^4}{4} + \dfrac{x^3}{3} \qquad (B)\ \dfrac{x^4}{4} + \dfrac{x^3}{3} + 3 \qquad (C)\ x^4 + \dfrac{x^3}{3} + 3$

$(D)\ \dfrac{x^4}{4} + x^3 \qquad (E)\ \dfrac{x^4}{4} + \dfrac{x^3}{3} + 1$

40. The absolute maximum and absolute minimum of $f(x) = x - \dfrac{9}{x}$ on$[1, 5]$ are at the points

(A) $(1, 10)$ and $(5, 6.8)$ (B) $(0, 10)$ and $(3, 6)$

(C) $(3, 6)$ and $(5, 6.8)$ (D) $(5, 6.8)$ and $(3, 6)$

(E) None of the above