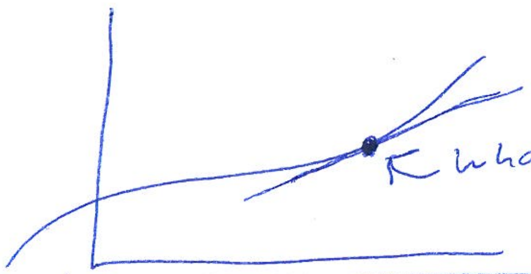
Section 2.4: Limits



← what's the slope?

maglev train's position: $s = f(t) = 4t^2$

ex $f(0) = 0 \quad f(1) = 4 \quad f(2) = 16 \quad f(3) = 36 \ \text{-----}$

how fast is it going at $t = 1$?

how can we get that from the information at hand?

$$\frac{f(3) - f(1)}{3 - 1} = \frac{36 - 4}{2} = \frac{32}{2} = 16$$

but can we be more accurate?

$$\frac{f(2) - f(1)}{2 - 1} = \frac{16 - 4}{1} = 12$$

even more?

$$\frac{f(1.5) - f(1)}{1.5 - 1} = \frac{9 - 4}{.5} = 10$$

$$\frac{f(1.01) - f(1)}{1.01 - 1} = \frac{4.0804 - 4}{.01} = 8.04$$

$$\frac{f(1.0001) - f(1)}{1.0001 - 1} = 8.0004$$

$$\boxed{g(x) = \frac{4x^2 - 4}{x - 1}}$$

the function f has a limit L as x approaches a

$$\lim_{x \to a} f(x) = L$$

if the value $F(X)$ can be made as close to L as we please by taking X sufficiently close to (but not equal to) a.

Ex evaluate $\displaystyle\lim_{x \to 1} \frac{4x^2-4}{x-1}$

$$= \lim_{x \to 1} \frac{4(x-1)(x+1)}{x-1} = \lim_{x \to 1} 4(x+1) = 8$$

what is the difference between

$$\frac{4x^2-4}{x-1} = \frac{4(x-1)(x+1)}{x-1} \quad \text{and}$$

$4(x+1)$?

domain? the hole at $x=1$?

EX

$$\lim_{h \to 0} \frac{\sqrt{1+h} - 1}{h}$$

$$\lim_{h \to 0} \frac{\sqrt{1+h} - 1}{h} \frac{(\sqrt{1+h} + 1)}{(\sqrt{1+h} + 1)}$$

$$\lim_{h \to 0} \frac{h}{h(\sqrt{1+h} + 1)}$$

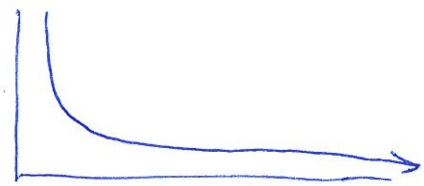
$$\lim_{h \to 0} \frac{1}{\sqrt{1+h} + 1} = \frac{1}{\sqrt{1} + 1} = \frac{1}{2}$$

---

how about as $x \to \infty$?

consider $1/x$

$\lim_{x \to \infty} 1/x = ?$



$$f(x) = \frac{2x^2}{1+x^2}$$

$f(x)$	1	1.6	1.92	1.98	1.9998	1.999998
x	1	2	5	10	100	1000

$\lim_{x \to \infty} f(x)$ seems to be heading towards 2 f has a limit as x increases without bound (x keeps approaching ∞)

Definition

$$\lim_{x \to \infty} f(x) = L$$

if f(x) can be made arbitrarily close to L by making x large enough

thrm

if $1/x^n$ is defined, then for all n,

$$\lim_{x \to \infty} 1/x^n = 0 \qquad \lim_{x \to -\infty} \frac{1}{x^n} = 0$$

reconsider $f(x) = \dfrac{2x^2}{1+x^2}$

we guessed (based on inductive reasoning) that $f(x) \to 2$ as $x \to \infty$. can we confirm?

trick determine the highest power of x, say 'n', multiply both the top and bottom by $1/x^n$

$$\lim_{x \to \infty} \frac{(2x^2)}{(1+x^2)} \frac{(1/x^2)}{(1/x^2)} = \lim_{x \to \infty} \frac{\frac{2x^2}{x^2}}{1/x^2 + x^2/x^2} = \lim \frac{2}{1/x^2 + 1} = \frac{2}{0+1} = 2$$

$$\lim_{x\to\infty} \frac{x^2 - x + 3}{2x^3 + 1} = \lim_{x\to\infty} \frac{x^2/x^3 - x/x^3 + 3/x^3}{2x^3/x^3 + 1/x^3} = \lim_{x\to\infty} \frac{1/x - 1/x^2 + 3/x^3}{2 + 1/x^3}$$

$$= \frac{0 - 0 + 0}{2 + 0} = 0$$

$\boxed{P5}$

$$\lim_{x\to\infty} \frac{2x^3 - 3x^2 + 1}{x^2 + 2x + 4} = \lim_{x\to\infty} \frac{2x^3 - 3x^2 + 1}{x^2 + 2x + 4} \frac{(1/x^3)}{(1/x^3)}$$

$$= \lim_{x\to\infty} \frac{2x^3/x^3 - 3x^2/x^3 + 1/x^3}{x^2/x^3 + 2x/x^3 + 4/x^3} = \lim_{x\to\infty} \frac{2 - 3/x + 1/x^3}{1/x + 2/x^2 + 4/x^3}$$

$$= \frac{2 - 0 + 0}{0 + 0 + 0} = 2/0 \qquad \text{DNE}$$