P2 Section 2.5: one sided limits P1

roperties of limits

thm

suppose $\lim_{x \to a} f(x) = L$ $\lim_{x \to a} g(x) = M$

then

1) $\lim_{x \to a} [f(x)]^r = [\lim_{x \to a} f(x)]^r = L^r$ $r > 0$

2) $\lim_{x \to a} c f(x) = c \lim_{x \to a} f(x) = cL$ $c \in \mathbb{R}$

3) $\lim_{x \to a} [f(x) \pm g(x)] = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x) = L \pm M$

4) $\lim_{x \to a} [f(x)\, g(x)] = [\lim_{x \to a} f(x)][\lim_{x \to a} g(x)] = LM$

5) $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)} = \frac{L}{M}$ if $M \neq 0$

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def

f has a right-hand limit L as x approaches a from the right,

$\lim_{x \to a^+} f(x) = L$

if the values of f(x) can be made as close to L as we please by taking x sufficiently close to (but not equal to) a (on the right hand side of a).

def

f has a left-hand limit L as x approaches a from the left,

$\lim_{x \to a^-} f(x) = M$

if the values of f(x) can be made ~~so~~ as close to L as we please by taking x sufficiently close to (but not equal to) a (on the right hand side).

thm let f be a function defined for all x values close to $x=a$ (but not necessarily for $x=a$ itself).

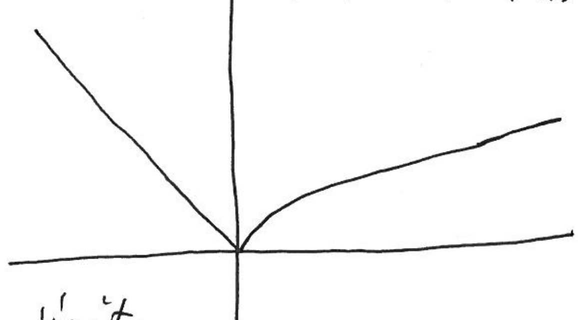
$$\lim_{x\to a} f(x) = L \quad \text{if and only if}$$

$$\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = L$$

Ex

$$f(x) = \begin{cases} -x & x \le 0 \\ \sqrt{x} & x > 0 \end{cases}$$

graph of $f(x)$



does $\lim_{x\to 0} f(x)$ exist?

let us consider each 1-sided limit.

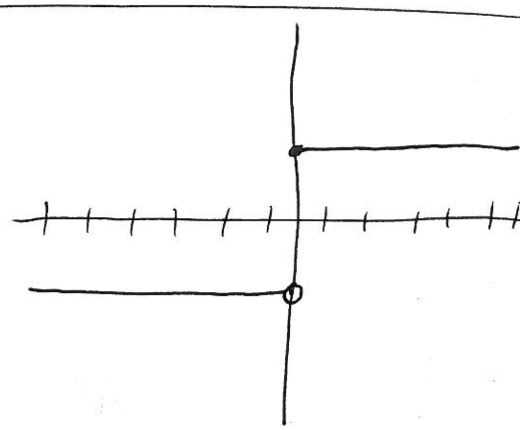
$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} (-x) = 0 \qquad \text{if } x \le 0$$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} (\sqrt{x}) = 0 \qquad \text{if } x > 0$$

$$\Rightarrow \lim_{x\to 0} f(x) = 0$$

Ex

$$g(x) = \begin{cases} -1 & x < 0 \\ 1 & x \ge 0 \end{cases}$$



does $\lim_{x\to 0} g(x)$ exist?

$$\lim_{x\to 0^-} g(x) = -1 \qquad \lim_{x\to 0^+} g(x) = 1$$

Def

f is continuous <u>at a number</u> $x = a$ if

a) $f(a)$ defined

b) $\lim_{x \to a} f(x)$ exists

c) $\lim_{x \to a} f(x) = f(a)$

f is continuous on an interval if f is continuous at each number in that interval

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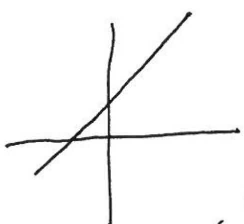
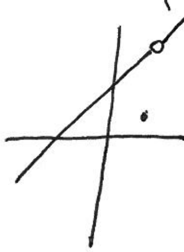
ex

$f(x) = x + 2$



continuous on $(-\infty, \infty)$

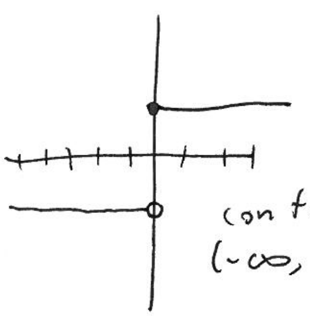
$$f(x) = \begin{cases} x+2 & x \neq 2 \\ 1 & x = 2 \end{cases}$$



discontinuous at $x = 2$

continuous on $(-\infty, 2) \cup (2, \infty)$

$$f(x) = \begin{cases} -1 & x < 0 \\ 1 & x \geq 0 \end{cases}$$



discontinuous at $x = 0$

continuous on $(-\infty, 0) \cup (0, \infty)$

Properties

a) $f(x) = c$ is continuous everywhere

b) $f(x) = x$ is continuous everywhere

if $f(x)$ and $g(x)$ are continuous at $x=a$

c) $[f(x)]^n$ $n \in \mathbb{R}$ is continuous at $x=a$ if it is defined at $x=a$

d) $f \pm g$ is continuous at $x=a$

e) fg is continuous at $x=a$

f) f/g is continuous at $x=a$ if $g(a) \neq 0$

thrm

1) Any polynomial $y = P(x)$ is continuous everywhere

2) any rational function $R(x) = P(x)/Q(x)$ is continuous for all values of x except where $Q(x) = 0$

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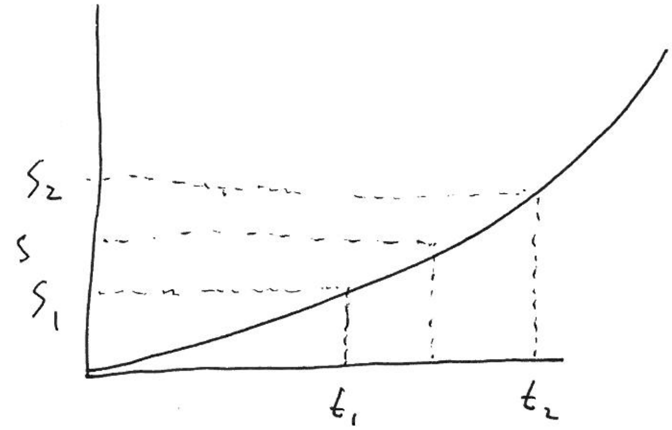
find values of the following for which they are continuous

$f(x) = 3x^3 + 2x^2 + x + 10$

all real numbers

$g(x) = \dfrac{8x^{10} - 4x + 1}{x^2+1}$

all real ~~[illegible]~~ ~~except from~~ ~~[illegible]~~.



if S_1 is a position at time t_1 and S_2 is another position at time t_2, then for any S between S_1 and S_2, there has to be atleast one t between t_1 and t_2 that corresponds to S

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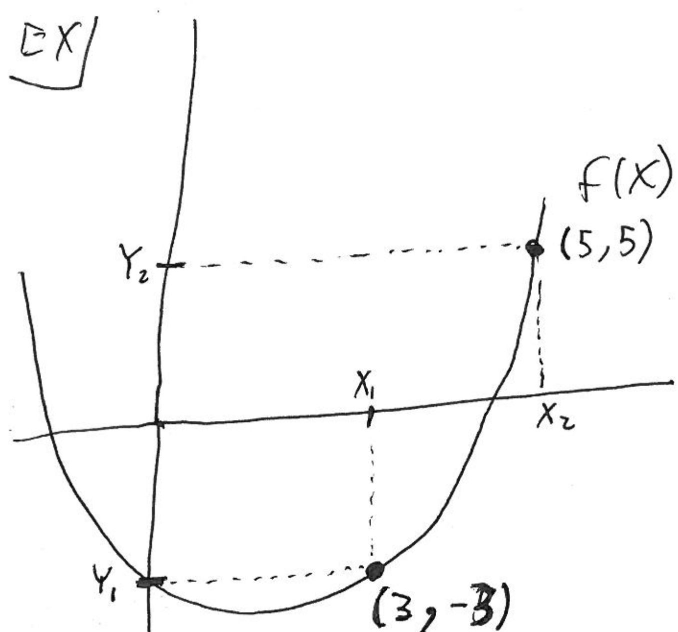
thrm

If f continoous on $[a, b]$ and M is between $f(a)$ and $f(b)$, then there is at least on number c in $[a, b]$ such that $f(c) = M$

EX

$$f(x) = (x-2)^2 - 4$$



there must be an x value corresponding to $Y=0$ that lies between X_1 and X_2