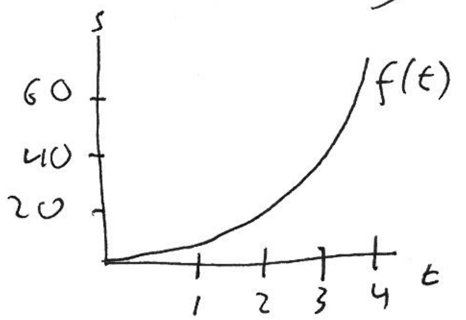
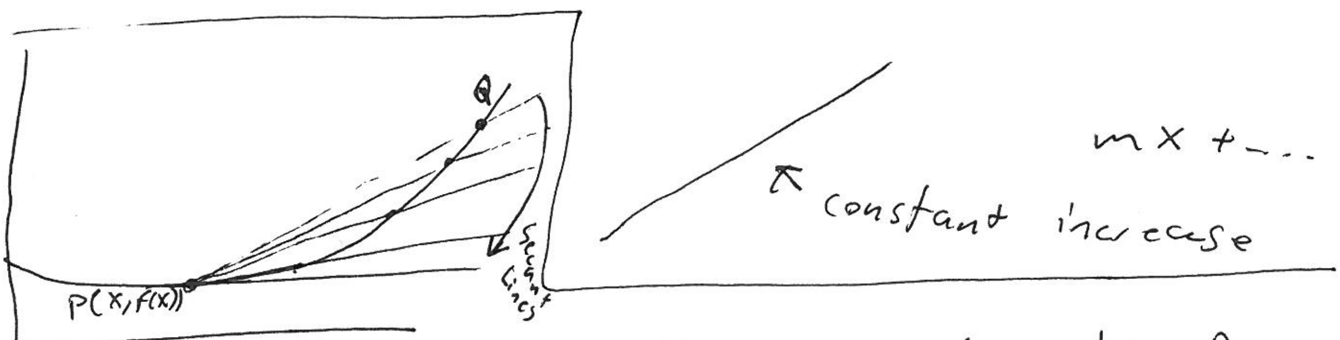
Section 2.6: derivative

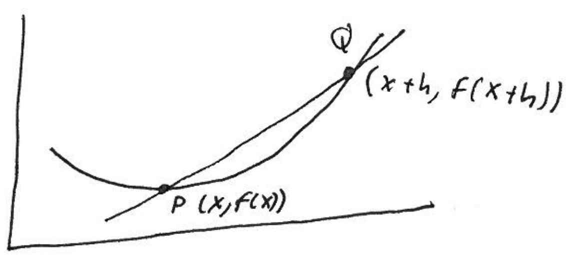
consider again $s = f(t) = 4t^2$



$ax^2 + \ldots$



$mx + \ldots$



Q moving closer to P does a better and better job approximating a tangent line of the graph of f at the point P

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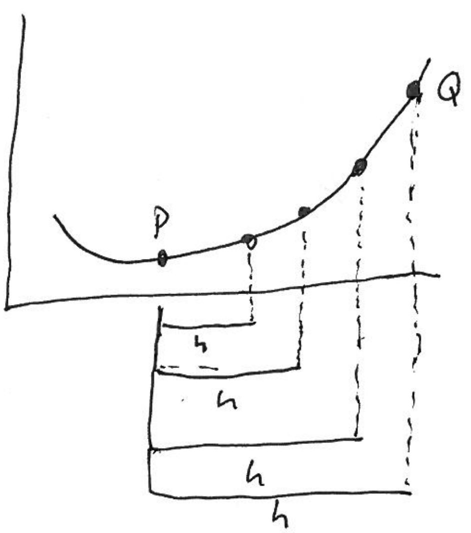
abuse of notation, but consider

$\lim_{h \to 0} Q = ?$ where does this limit head? P?



$$\lim_{h \to 0} Q = P$$

$\boxed{P2}$ $m = \dfrac{Y_2 - Y_1}{X_2 - X_1}$ how do we express Y_1, Y_2?

$Y = f(X)$, so $Y_1 = f(X_1)$ and $Y_2 = f(X_2)$

we can now write

$$m = \frac{f(X_2) - f(X_1)}{X_2 - X_1}$$

but we are interested in the slope between p and Q, and the coordinates for Q are $Y_2 = f(x+h)$ $X_2 = X+h$ and for P, $Y_1 = f(X)$ and $X_1 = X$

so now slope is

$$m = \frac{f(x+h) - f(x)}{(x+h) - x} = \frac{f(x+h) - f(x)}{h}$$

as we disgused before we're going to see what happens as $Q \to P$, that is, as $h \to 0$

$$\lim_{h \to 0} \underbrace{\frac{f(x+h) - f(x)}{h}}_{\text{differance quotient}}$$

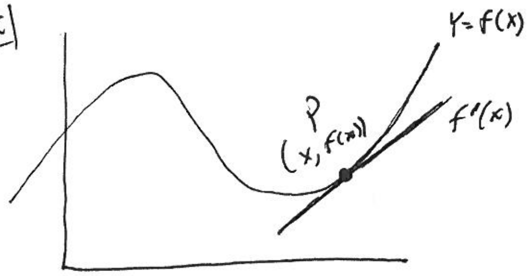
← instantaneous rate of change or slope of tangent line the derivative of a function f with respect to x is the function f' ("f prime")

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

domain: all real numbers for which the limit exists

$f'(x)$ gives the slope of the tangent line of f at any point $(x, f(x))$

Ex



other notations:

$D_x f(x)$ "d sub x of f of x"

if the function is ~~written~~ $y=f(x)$:

$\frac{dy}{dx}$ "d y d x"

y' "y prime"

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to find $f'(x)$

- find $f(x+h)$
- find $[f(x+h)] - f(x)$
- simplify $\frac{[f(x+h) - f(x)]}{h}$
- compute $\lim_{h \to 0} \left[\frac{f(x+h) - f(x)}{h}\right] = f'(x)$

EX find the slope of the tangent line of $f(x) = 3x+5$ at any point

$f(\_) = 3(\_) + 5$

* $f(x+h) = 3(x+h) + 5 = 3x+3h+5$

* $[f(x+h)] - f(x) = [3x+3h+5] - (3x+5)$

$= 3x - 3x + 3h + 5 - 5$

$= 3h$

* $\dfrac{[f(x+h) - f(x)]}{h} = \dfrac{3h}{h}$

* $\lim_{h \to 0} \left[\dfrac{f(x+h) - f(x)}{h} \right] = \lim_{h \to 0} \dfrac{3h}{h} \dfrac{(1/h)}{(1/h)} = \lim_{h \to 0} \dfrac{3}{1} = 3$

EX $f(x) = x^2$

$f(\_\_) = (\_\_)^2$

* $f(x+h) = (x+h)^2 = x^2 + 2xh + h^2$

* $[f(x+h)] - f(x) = [x^2 + 2xh + h^2] - x^2$

$= 2xh + h^2$

* $\dfrac{[f(x+h) - f(x)]}{h} = \dfrac{2xh + h^2}{h}$

$= \dfrac{\cancel{h}(2x+h)}{\cancel{h}}$

* $\lim_{h \to 0} 2x + h = 2x = f'(x)$