Section 2.6 + 3.1

Ex
$f(x) = 1/x$

(a) find $f'(x)$
(b) Slope of tangent line to f if $x=1$
(c) equation of tangent line from (b)

(a)
$f(x+h) = \frac{1}{x+h}$

$f(x+h) - f(x) = \frac{1}{x+h} - \frac{1}{x} = \frac{x}{x(x+h)} - \frac{(x+h)}{x(x+h)} = \frac{-h}{x(x+h)}$

$\frac{x-(x+h)}{x(x+h)} = \frac{x-x-h}{x(x+h)}$

$\frac{f(x+h) - f(x)}{h} = \frac{-\cancel{h}}{x(x+h)} \frac{1}{\cancel{h}} = \frac{-1}{x(x+h)}$

$\lim_{h \to 0} \frac{-1}{x(x+h)} = -\frac{1}{x^2} = f'(x)$

(b) $f'(1) = -1/(1)^2 = -1$

(c) $f(1) = 1/1$ so point $(1,1)$ is on tangent line

$m = f'(1) = -1$

$Y = -1x + b$

$1 = -1(1) + b$

$b = 2$

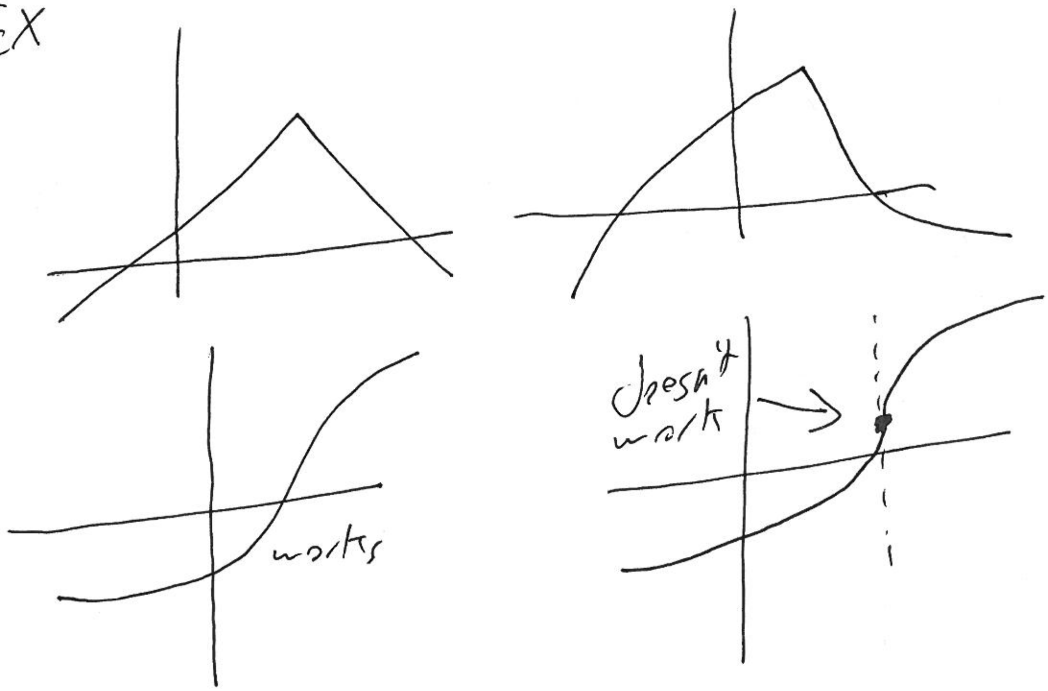
$Y = -x + 2$

differentiability

functions are not diferentiable if

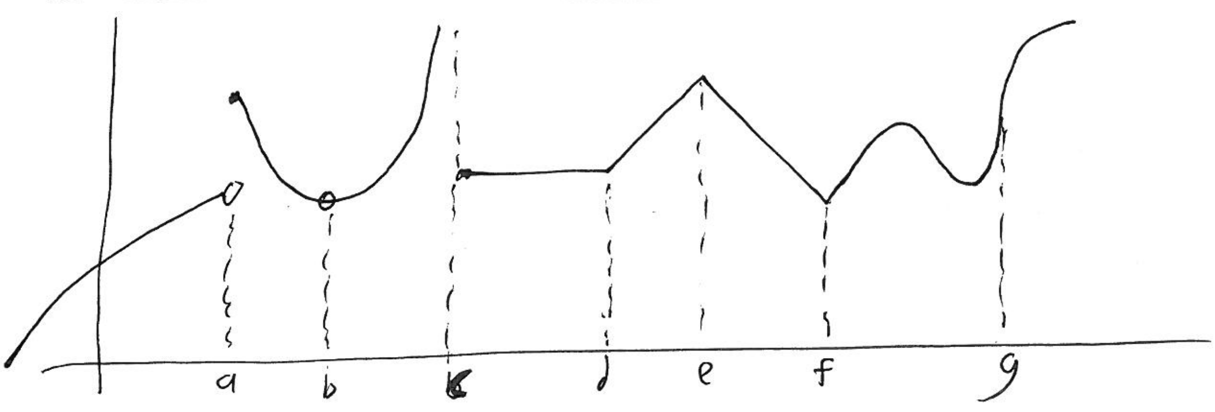
* the graph of f makes an abrupt chang (called a "corner")
* at a point where the tangent line is verticle (has undefined slope)

EX



if a function is differentiable at $x=a$, then it is continuous at $x=a$

continuity is necessary but not sufficient for a function to be differentiable



Section 3.1

P3

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$\downarrow$

$\frac{d}{dx}[f(x)]$ "d, dx of f of x"

derivative of f with respect to x, taken at x

rule 1 $\quad \frac{d}{dx}[c] = 0$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{c - c}{h} = 0$$

rule 2 $\quad \frac{d}{dx}[x^n] = n x^{n-1}$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

for $f(x) = x^2$

$$\lim_{h\to 0} \frac{(x+h)^2 - x^2}{h} = \lim_{h\to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{x^2}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(2x+h)}{\cancel{h}} = 2x$$

Ex $f'(x) = ?$ if

$f(x) = x$ $\quad$ $f(x) = x^8$ $\quad$ $f(x) = x^{5/2}$

$f'(x) = 1x^0 = 1$ $\quad$ $f'(x) = 8x^7$ $\quad$ $f(x) = \frac{5}{2} x^{5/2 - 1}$

$= \frac{5}{2} x^{3/2}$

EX $f'(x) = ?$

$$f(x) = \sqrt{x}$$
$$f(x) = x^{1/2}$$
$$f'(x) = \tfrac{1}{2} x^{1/2 - 1}$$
$$= \tfrac{1}{2} x^{-1/2}$$
$$= \frac{1}{2\sqrt{x}}$$

.7

$$f(x) = \frac{1}{\sqrt[3]{x}}$$
$$= x^{-1/3}$$
$$f'(x) = -\tfrac{1}{3} x^{-1/3 - 1}$$
$$= -\tfrac{1}{3} x^{-4/3}$$
$$= \frac{-1}{3x^{4/3}}$$

rule

$$\frac{d}{dx}\left[c f(x)\right] = c \frac{d}{dx}\left[f(x)\right]$$

let
$$g(x) = c f(x)$$
$$g'(x) = \lim_{h \to 0} \frac{g(x+h) - g(x)}{h}$$
$$= \lim_{h \to 0} \frac{c f(x+h) - c f(x)}{h}$$
$$= \lim_{h \to 0} c\left[\frac{f(x+h) - f(x)}{h}\right]$$
$$= c \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$
$$= c f'(x)$$

EX)
$$\frac{d}{dx}\left[5x^3\right]$$
$$= 5 \frac{d}{dx}\left[x^3\right]$$
$$= 5(3x^2)$$
$$= 15x^2$$

P5

rule 4

$$\frac{d}{dx}\left[f(x) \pm g(x)\right]$$
$$= \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)]$$

let $\quad s(x) = f(x) \pm g(x)$

$$\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}[s(x)] = s'(x)$$

$$= \lim_{h \to 0} \frac{s(x+h) - s(x)}{h}$$

$$= \lim_{h \to 0} \frac{[f(x+h) \pm g(x+h)] - [f(x) \pm g(x)]}{h}$$

$$= \lim_{h \to 0} \frac{f(x+h) \pm g(x+h) - f(x) - (\pm g(x))}{h}$$

$$= \lim_{h \to 0} \frac{f(x+h) - f(x) \pm [g(x+h) - g(x)]}{h}$$

$$= \lim_{h \to 0} \left(\frac{f(x+h) - f(x)}{h} \pm \frac{g(x+h) - g(x)}{h}\right)$$

$$= \left[\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}\right] \pm \left[\lim_{h \to 0} \frac{g(x+h) - g(x)}{h}\right]$$

$$= f'(x) \pm g'(x)$$

$$= \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)]$$