Section 3.3: chain rule

consider $h(x) = (x^2 + x + 1)^2$

what is $h'(x)$?

	x^2	x	1
x^2	x^4	x^3	x^2
x	x^3	x^2	x
1	x^2	x	1

$= x^4 + 2x^3 + 3x^2 + 2x + 1$

so $h'(x) = 4x^3 + 6x^2 + 6x + 2$

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what if $h(x) = (x^2 + x + 1)^{100}$ instead?
how do we find $h'(x)$

$h(x)$ is a composite function

if $f(x) = x^{100}$ and
$g(x) = x^2 + x + 1$, then
$h(x) = f(g(x))$

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as a thought experiment, consider some arbitrary
$h(x) = f(g(x))$
let $u = f(x)$ and $y = g(f(x)) = g(u)$
imagine that, because of $f(x)$, u changes 3 times as fast as x
also, because of $g(u)$, y changes 4 times as fast as u

~~h(x)=~~

in other words

$$f'(x) = \frac{du}{dx} = 3$$

$$g'(u) = \frac{dy}{du} = 4$$

$$h'(x) = 4 * 3 = \underset{g'(u)}{\frac{dy}{du}} \; \underset{f'(x)}{\frac{du}{dx}} = 12$$

<u>chain rule</u> $\quad h(x) = g(u) \quad u = f(x)$

$$h'(x) = \frac{d}{dx}\left[g(f(x))\right] = g'(f(x))\, f'(x) = g'(u)\, f'(x)$$

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to begining example

$$h(x) = (x^2 + x + 1)^{100}$$

$$u = f(x) = x^2 + x + 1 \qquad g(u) = u^{100}$$

$$g'(u) = 100\, u^{99}$$

$$f'(x) = 2x + 1$$

$$h'(x) = g'(u)\, f'(x) = 100(x^2 + x + 1)^{99}(2x + 1)$$

EX $h(x) = \sqrt{x^2+1} = (x^2+1)^{1/2}$

So $g(u) = x^{1/2}$ if $u = f(x) = x^2+1$

$$h'(x) = g'(u)\, f'(x)$$

$$= \tfrac{1}{2} u^{-1/2}\, 2x$$

$$= \tfrac{1}{2} (x^2+1)^{-1/2}\, 2x$$

$$= \frac{2x}{2\sqrt{x^2+1}}$$

$$= \frac{x}{\sqrt{x^2+1}}$$

EX) $h(x) = (2x^2+3)^4 (3x-1)^5$

chain rule

$$h'(x) = (2x^2+3)^4 \frac{d}{dx}\left[(3x-1)^5\right] + \frac{d}{dx}\left[(2x^2+3)^4\right](3x-1)^5$$

$$= (2x^2+3)^4\left[5(3x-1)^4 3\right] + \left[4(2x^2+3)^3(4x)\right](3x-1)^5$$

$$= 15(2x^2+3)^4(3x-1)^4 + 16(x)(2x^2+3)^3(3x-1)^5$$

Ex

$$h(x) = \frac{1}{(4x^2-7)^2}$$

$$= (4x^2-7)^{-2}$$

$g(u) = u^{-2}$ $\quad$ $f(x) = 4x^2 - 7$