Section 3.6

everything so far has been

$$Y = f(X)$$

what if we have something like

$$X^2Y + Y - X^2 + 1 = 0$$

there isn't a way to express Y explicitly in terms of X.
this is an implicit equation

this can easily be made an explicit equation:

$$Y(X^2+1) = X^2-1$$

$$Y = \frac{X^2-1}{X^2+1}$$

what if we have something more complicated? eg

$$Y^4 - Y^3 - Y + 2X^3 - X = 8$$

how to find an explicit equation

how do we take the derivative?

implicit differentiation

EX

$$Y^2 = X$$

differentiate both sides (take d/dx)

$$\frac{d}{dx}\left[Y^2\right] = \frac{d}{dx}[X]$$

but we know that $Y = f(X)$, so

$$\frac{d}{dX}\left[\left(f(X)\right)^2\right] = \frac{d}{dx}[X]$$

$$2\left(f(X)\right)f'(X) = 1$$

$$2Y\frac{dY}{dX} = 1$$

Solve for dY/dx

$$\frac{dY}{dX} = \frac{1}{2Y}$$

Steps

1) differentiate both sides with respect to X

2) solve for dY/dx

find $\partial y/\partial x$ of $y^3 - y + 2x^3 - x = 8$

$\boxed{EX}$

$$\frac{\partial}{\partial x}\left[y^3 - y + 2x^3 - x\right] = \frac{\partial}{\partial x}[8]$$

$$\frac{\partial}{\partial x}[y^3] - \frac{\partial}{\partial x}[y] + \frac{\partial}{\partial x}[2x^3] - \frac{\partial}{\partial x}[x] = 0$$

$y = f(x)$

$$\frac{\partial}{\partial x}\left[(f(x))^3\right] - \frac{\partial}{\partial x}\left[(f(x))\right] + 6x^2 - 1 = 0$$

$$3(f(x))^2 f'(x) - f'(x) + 6x^2 - 1 = 0$$

$$3y^2 \frac{\partial y}{\partial x} - \frac{\partial y}{\partial x} = 1 - 6x^2$$

$$(3y^2 - 1)\frac{\partial y}{\partial x} = 1 - 6x^2$$

$$\frac{\partial y}{\partial x} = \frac{1 - 6x^2}{3y^2 - 1}$$

$\boxed{EX}$ $X^2 + Y^2 = 4$

a) find dY/dX

b) find slope of tangent line at $(1, \sqrt{3})$

a) $$\frac{d}{dX}\left[X^2 + Y^2\right] = \frac{d}{dX}[4]$$

$$\frac{d}{dX}\left[X^2\right] + \frac{d}{dX}\left[Y^2\right] = 0$$

$$2X + 2Y\frac{dY}{dX} = 0$$

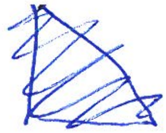
$$2Y\frac{dY}{dX} = -2X$$

$$\frac{dY}{dX} = \frac{-2X}{2Y} = \frac{-X}{Y}$$

b) $$\left.\frac{dY}{dX}\right|_{(1,\sqrt{3})} = \left.-\frac{X}{Y}\right|_{(1,\sqrt{3})} = -\frac{(1)}{(\sqrt{3})} = -1/\sqrt{3}$$

$\left.\frac{dY}{dX}\right|_{(a,b)}$ means: evaluate the derivative dY/dX at the point (a, b)

EX #10

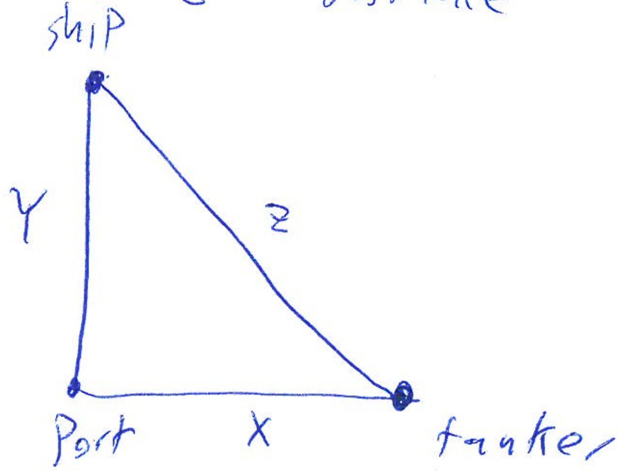


P5

x = distance tanker $\Leftrightarrow$ port

y = distance ship $\Leftrightarrow$ port

z = distance ship $\Leftrightarrow$ tanker



$$z^2 = x^2 + y^2$$

want to find $\frac{dz}{dt}$

$$\frac{d}{dt}\left[z^2\right] = \frac{d}{dt}\left[x^2 + y^2\right]$$

$$2z\frac{dz}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$$

$$z\frac{dz}{dt} = x\frac{dx}{dt} + y\frac{dy}{dt}$$

$$z^2 = (30)^2 + (40)^2 = 2500 \qquad z = 50$$

$$50\frac{dz}{dt} = (30)(20) + (40)(30) \Rightarrow \frac{dz}{dt} = 36$$