Section 3.7

Def | $\Delta x = x_2 - x_1$ increment in x

EX $x_1 = 3$ $x_2 = 3.2$

$$\Delta x = x_2 - x_1 = .2$$

note: Δx is similar to h in the differance quotient

Def | $\Delta y = f(x + \Delta x) - f(x)$

increment in y

EX $f(x) = x^3$ $x_1 = 2$ $x_2 = 2.01$

$\Delta x = .01$

$\Delta y = (2.01)^3 - (2)^3 = 8.120601 - 8 = 0.120616$

Close to the tangent line, the tangent line approximates the function, in other words, dy approximates Δy

slope of tangent line is given by $\frac{dy}{\Delta x}$ also $f'(x)$

$$\frac{dy}{\Delta x} = f'(x) \rightarrow dy = f(x)\Delta x$$

$$dy = f'(x)\,dx$$

differential dx
$dx = \Delta x$
differential dy
$dy = f'(x)\Delta x = f'(x)\,dx$

EX let $Y = X^3$

a) find the differential dY of Y

b) use dY to approximate ΔY when X changes from 2 to 2.01

b) use dY to approximate ΔY when X changes from 2 to 1.98

a) $dY = f'(x)\,dx = 3x^2\,dx$

b) $X = 2 \qquad dx = 2.01 - 2 = .01$

$dY = 3x^2\,dx = 3(2)^2(.01) = .12$

c) $X = 2 \qquad dx = 1.98 - 2 = -.02$

$dY = 3x^2\,dx = 3(2)^2(-.02) = -.24$

EX approximate the value of $\sqrt{26.5}$ using differentials

consider $Y = f(x) = \sqrt{x}$

25 is the nearest number to 26.5 for which we know the square root

$dY = f'(x)\,dx = \frac{1}{2}(x)^{-1/2}\,dx$

$dY = \frac{1}{2}(25)^{-1/2}(1.5) = \frac{1}{2}(5)^{-1}(1.5) = \frac{1.5}{10} = 0.15$

if $Y = 5$, then $dY + Y = 5.15$

so $\sqrt{26.5} \approx 5.15$

Ex 5 cost of operating a truck on a 500 mi trip is given by

$$C(V) = 125 + V + \frac{4500}{V}$$

where V is the average speed traveled

find the approximate chang in cost when the average speed changes from 55 to 58

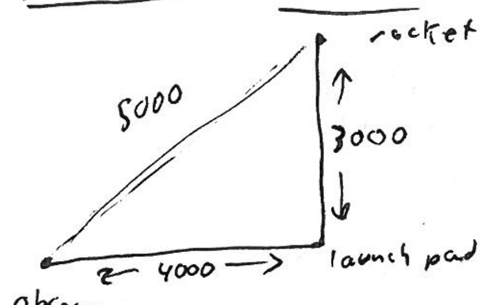
$$dC = C'(V)\,dV \approx \Delta C$$

$$= \left(1 - \frac{4500}{V^2}\right) dV$$

$$dC = \left(1 - \frac{4500}{(55)^2}\right)(3) = \left(1 - \frac{4500}{3025}\right)(3) = -1.46$$

cost decreases by \$1.46

P 223 **EX 9** (§3.6)



x = distance between rocket and spectator

Y = rocket altitude

$Y = 3000 \quad \frac{dY}{dt} = 600$

$x^2 = Y^2 + 4000^2 \qquad X = 5000$

$$\frac{d}{dt}[x^2] = \frac{d}{dt}[Y^2 + 4000^2] \rightarrow 2x\frac{dx}{dt} = 2Y\frac{dY}{dt} \rightarrow x\frac{dx}{dt} = Y\frac{dY}{dt}$$

$$5000\frac{dx}{dt} = 3000(600) \rightarrow \frac{dx}{dt} = \frac{3000(600)}{5000} = 360$$