Section 3.4

def **marginal cost:** the cost of producing one additional item

$C(x)$ = cost function

$C'(x)$ = marginal cost

$\overline{C(x)} = \frac{C(x)}{x}$ = average cost

$\overline{C'(x)}$ = marginal average cost

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if $p = f(x)$ is the price, and x is units sold or demand, then revenue is

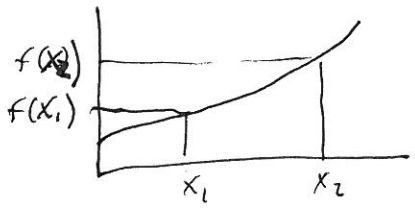
$R(x) = p\,x$

profit is the difference between revenue and costs

$P(x) = R(x) - C(x)$

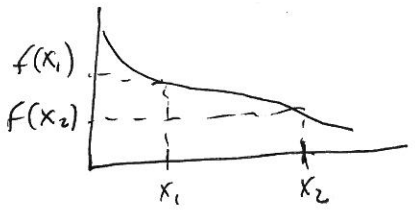
Section 4.1

increasing



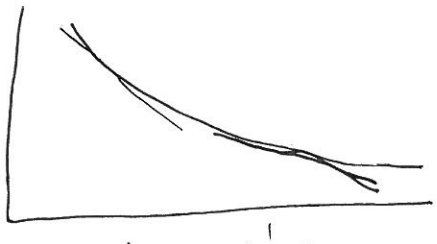
if, for all numbers in (a, b) $f(x_1) < f(x_2)$ whenever $x_1 < x_2$, then $f(x)$ is increasing on (a, b)

Decreasing

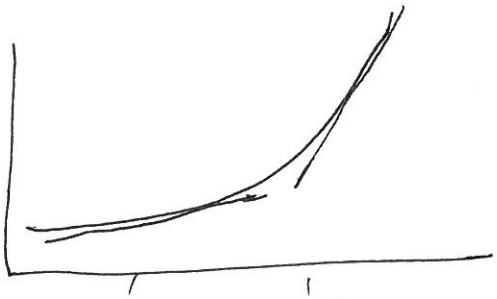


if, for all numbers in (a, b) $f(x_1) > f(x_2)$ whenever $x_1 < x_2$, then $f(x)$ is decreasing on (a, b)

what slope do we associate with increasing? decreasing?



decreasing
negative slope



Increasing
positive slope

thm 1

if $f'(x) > 0$ for all values of x in (a,b)
then $f(x)$ is increasing on (a,b)
if $f'(x) < 0$ for all values of x in (a,b)
then $f(x)$ is decreasing on (a,b)
if $f'(x) = 0$ for all values of x in (a,b)
then $f(x)$ is constant on (a,b)

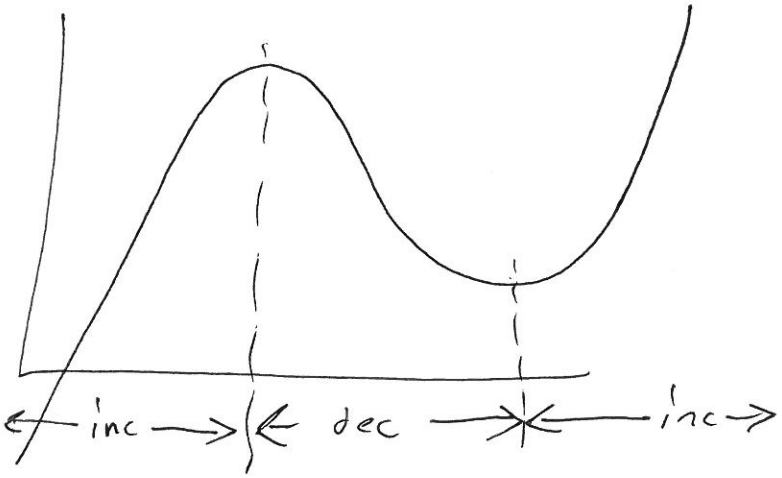
EX find where $f(x) = x^2$ is increasing and decreasing

$f'(x) = 2x$

increasing if
$f'(x) = 2x > 0$
$x > 0$
$(0, \infty)$

decreasing if
$f'(x) = 2x < 0$
$x < 0$
$(-\infty, 0)$

technique



what happens when it changes from increasing to decreasing or from decreasing to increasing?

$m = f'(x) = 0$

* find all values of $f'(x) = 0$ or f' discontinuous and find the open intervals determined by these values
* select a test value c in each interval and test the sign of the derivative at that point $f'(c)$
  a. if $f'(c) > 0$, f increasing on that interval
  b. if $f'(c) < 0$, f decreasing on that interval

P4/

EX $f(x) = x^3 - 3x^2 - 24x + 32$

$f'(x) = 3x^2 - 6x - 24$

$= 3(x+2)(x-4)$

zeros: $x = -2 \quad x = 4$

intervals: $(-\infty, -2) \quad (-2, 4) \quad (4, \infty)$

interval	test point c	f'(c)	sign	inc?
$(-\infty, -2)$	-3	21	+	✓
$(-2, 4)$	0	-24	−	x
$(4, \infty)$	5	21	+	✓

increasing: $(-\infty, -2) \cup (4, \infty)$

decreasing: $(-2, 4)$

EX $f(x) = x^{2/3} = \sqrt[3]{x^2}$

find intervals of inc/dec

$f'(x) = \frac{2}{3}x^{-1/3} = \frac{2}{3x^{1/3}}$

$f'(x)$ undefined at $x = 0$ so discontinuous at $x = 0$

$f'(x)$ has no zeros

intervals: $(-\infty, 0) \quad (0, \infty)$

interval	test point c	f'(c)	sign	inc
$(-\infty, 0)$	-1	$-2/3$	−	x
$(0, \infty)$	1	$2/3$	+	✓

increasing: $(0, \infty)$

decreasing: $(-\infty, 0)$