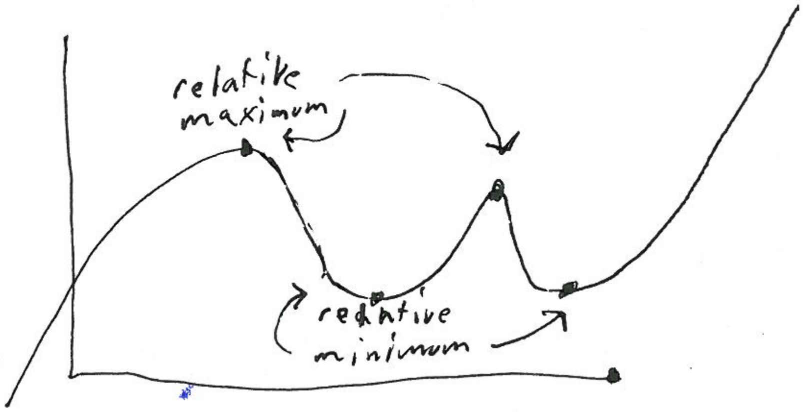
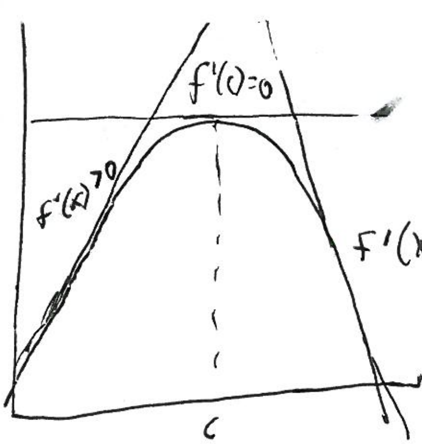
Section 4.1
continued

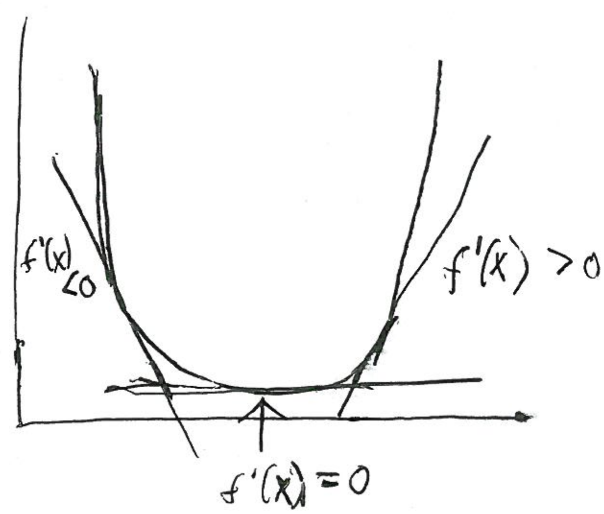


each of these points are extreme values only when compared to nearby values, thus relative or local extrema

Def a function f has relative maximum at $x=c$, if there exists an open interval (a,b) containing c such that $f(x) \leq f(c)$ for all x in (a,b)

Def a function f has relative minimum at $x=c$ if there exists an open interval (a,b) containing c such that $f(x) \geq f(c)$ for all x in (a,b)

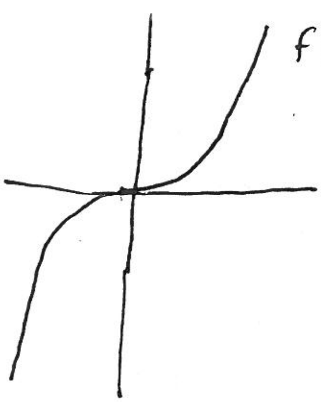




P2

it seems that if we have a maximum or a minimum, then $f'(c)=0$

but if $f'(c)=0$, do we always have a max/min?



$f'(x)=2x^2$

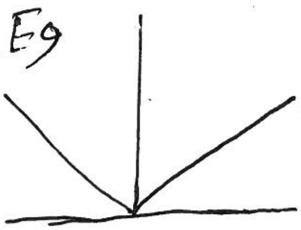
$0=2x^2$

$x=0$

but in this case we dont have either a max or a min when $x=0$

this observation also doesn't help us to find max/min values when f isn't differentiable

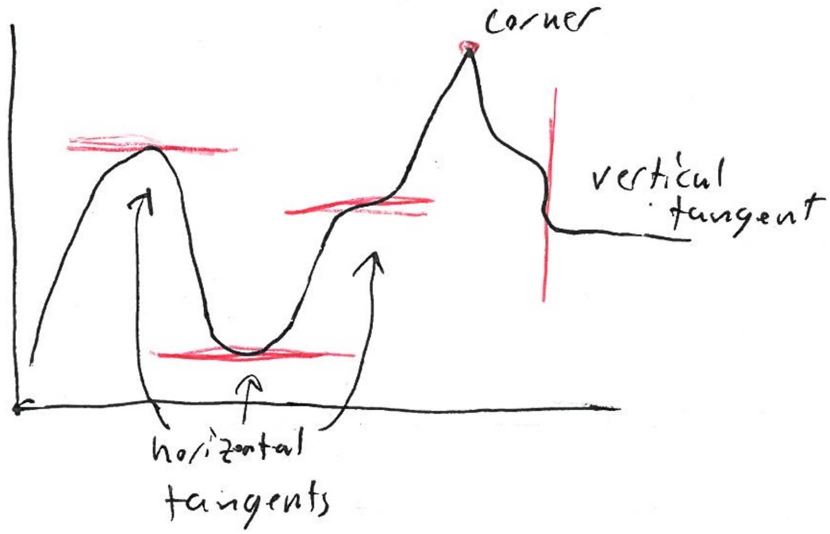
Eg



corner $\Rightarrow$ not diferentiable @ $x=0$ but clearly a minimum at $x=0$

Def | a critical number/critical value of a function f is any number x in the domain of f such that $f'(x)=0$ of $f'(x)$ does not exist.

EX



$\boxed{\text{critical values}}$

* to find relative extrema of continuous function f

- find <u>critical values</u> of f
- determine sign of $f'(x)$ to the left and right of each critical value
    - if positive → negative f has a relative max at $x=c$
    - if negative → positive f has a relative min at $x=c$
    - if does not change, then not an extrema at $x=c$

<u>EX</u> Find relative max/min of $f(x)=x^2$

$f'(x) = 2x$

$0 = 2x \Rightarrow x = 0$

to the left of $x=0$, $f'(x) < 0$

to the right of $x=0$, $f'(x) > 0$

negative → positive

$\Rightarrow$ relative min at $x=0$

P4

EX

find relative maximum and minimum of

$$f(x) = x + 1/x$$

$$f'(x) = 1 + -1/x^2 = 1 - \frac{1}{x^2} = \frac{x^2-1}{x} = \frac{(x+1)(x-1)}{x^2}$$

~~$f'(x)$ has no solution~~

$f'(x)$ undefined for $x = 0$

$$0 = \frac{(x^2-1)(x+1)}{x^2} \rightarrow 0 = (x-1)(x+1)$$

$x = 1,\ x = -1$

~~[illegible]~~ $x = 0, 1, -1$

although $f(0)$ also undefined, so no possible max or min there

critical values: $x = 1, -1$

$f'(x)$

$f'(-1.5) = .\overline{5}$ $f'(-.5) = -3$ $f'(.5) = -3$ $f'(1.5) = .\overline{5}$

-1.5   -1   -.5   .5   1   1.5

+ + + +   - - - - - - -   + + + +

positive → negative   negative → positive

relative max   relative min