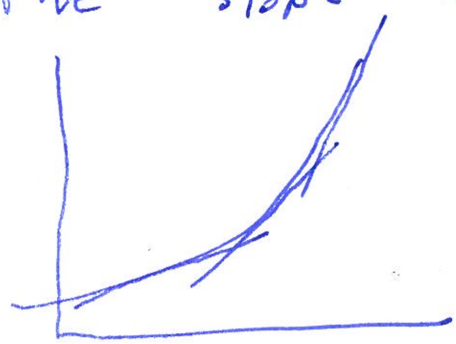
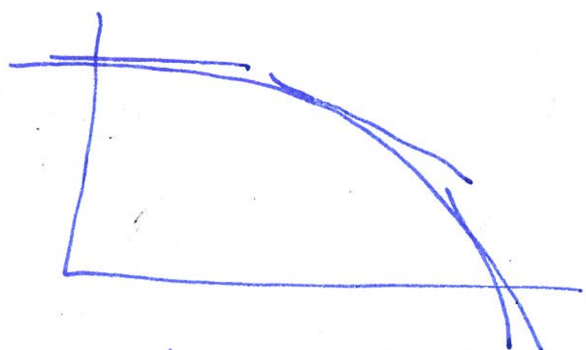
2015-03-08

P1

4.2 2nd derivative

2nd derivative gives the rate of change of ~~the the tangent~~ $f'(x)$, or the rate of change of the slope of the tangent line





~~increasing~~ slope
slope of tangent line increasing

slope of tangent line decreasing

$f''(x) > 0 \Rightarrow f'(x)$ increasing

$f''(x) < 0 \Rightarrow f'(x)$ decreasing

Def

$f(x)$ is <u>concave up</u> on (a,b) if $f'(x)$ is increasing on (a,b) [$f''(x) > 0$ for all x in (a,b)]

$f(x)$ is <u>concave down</u> on (a,b) if $f'(x)$ is decreasing on (a,b) [$f''(x) < 0$ for all x in (a,b)]

concave up

concave down

p2

to determine concavity

- identify "critical values" of $f''(x)$ ($f''(x)=0$ or $f''(x)$ undefined) and determine the associated open intervals
- determine sign of $f''(x)$ on each of the intervals
  - $f''(x) > 0 \Rightarrow$ concave up
  - $f''(x) < 0 \Rightarrow$ concave down

EX

$$f(x) = x^3 - 3x^2 - 24x + 32$$

$$f'(x) = 3x^2 - 6x - 24$$

$$f''(x) = 6x - 6$$

$$f''(x) = 0 \Rightarrow \quad 0 = 6x - 6$$

$$x = 1$$

	test value c	$f''(c)$	sign	concavity
$(-\infty, 1)$	0	-6	$-$	down
$(1, \infty)$	2	6	$+$	up

concave up on $(1, \infty)$

concave down on $(-\infty, 1)$

---

inflection point: a point on a continuous f where the tangent line exists and the concavity changes

steps to find inflection points

- compute $f''(x)$
- find c such that $f''(c) = 0$ or $f''(c)$ dne
- determine sign of points from above if $f''(x)$ changes sign as we cross $x=c$, then $(c, f(c))$ is an inflection point

EX) Find inflection point of

$f(x) = x^3$

$f'(x) = 2x^2$

$f''(x) = 6x$

$x = 0$ is our "critical value"

we go from $-$ to $+$ so we have a change in sign and $x = 0$ is an inflection point

---

EX) determine where $f(x)$ is concave up/down and find the points of inflection

if $f(x) = \dfrac{1}{x^2+1}$

$= (x^2+1)^{-1}$

$$f'(x) = -(x^2+1)^{-2}(2x) = \frac{-2x}{(x^2+1)^2}$$

$$f''(x) = \frac{(x^2+1)^2(-2) + (2x)(2)(x^2+1)(2x)}{(x^2+1)^4}$$

$$= \frac{(x^2+1)(6x^2-2)}{(x^2+1)^4} = \frac{2(3x^2-1)}{(x^2+1)^3}$$

continuous everywhere, zero if

$3x^2 - 1 = 0$ or $x^2 = 1/3$ or $x = \pm 1/\sqrt{3}$

intervals: $(-\infty, -1/\sqrt{3})$ $(-1/\sqrt{3}, 1/\sqrt{3})$ $(1/\sqrt{3}, \infty)$ $\boxed{P4}$

	test value	$f''(c)$	concavity	
$(-\infty, -1/\sqrt{3})$	-1	$1/2$	up	
$(-1/\sqrt{3}, 1/\sqrt{3})$	0	-2	down	$+ \to -$
$(1/\sqrt{3}, \infty)$	1	$1/2$	up	$- \to +$

inflection points: $x = \pm 1/\sqrt{3}$
need y values

$f(-1/\sqrt{3}) = 3/4$ $\quad$ $f(1/\sqrt{3}) = 3/4$

Points: $(-1/\sqrt{3}, 3/4)$ $(1/\sqrt{3}, 3/4)$

concave up: $(-\infty, -1/\sqrt{3}) \cup (1/\sqrt{3}, \infty)$
concave down: $(-1/\sqrt{3}, 1/\sqrt{3})$

EX 1 sketch a graph w/ the following properties

$f(-1) = 4$ $\quad$ $f(1) = 0$ $\quad$ $f'(-1) = 0$

$f(0) = 2$ $\quad$ $f'(1) = 0$

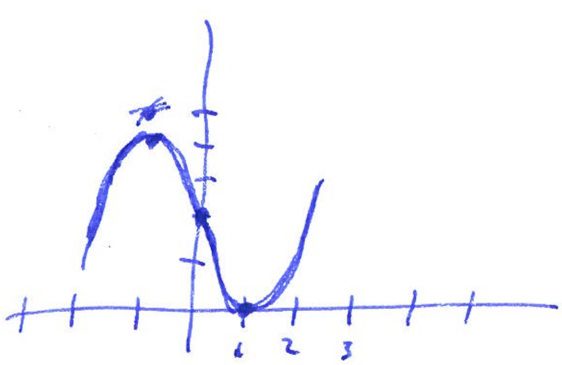
~~$f(1) = 2$~~

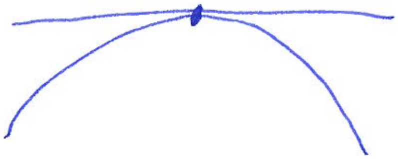
$f'(x) > 0$ on $(-\infty, -1)$ and $(1, \infty)$

$f'(x) < 0$ on $(-1, 1)$

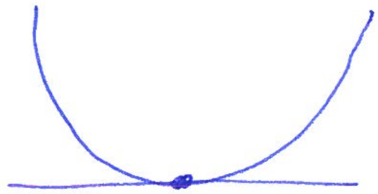
$f''(x) < 0$ on $(-\infty, 0)$

$f''(x) > 0$ on $(0, \infty)$





max
concave down



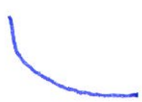
min
concave up

2nd derivative test

- compute $f'(x)$, $f''(x)$
- find critical values of f where $f'(x)=0$
- compute $f''(x)$ for each critical value c
    - if $f''(c)<0$ then f has max at c
    - if $f''(c)>0$ then f has min at c
    - if $f''(c)=0$ or $f''(c)$ dne, then inconclusive (switch to 1st derivative test)

- $f'(x)>0$ increasing
  $f''(x)>0$ concave up 

- $f'(x)>0$ increasing
  $f''(x)<0$ concave down 

- $f'(x)<0$ decreasing
  $f''(x)>0$ concave up 

- $f'(x)<0$ decreasing
  $f''(x)<0$ concave down