Section 4.4

Def | if $f(x) \le f(c)$ for all x in the domain of f, then $f(c)$ is an absolute maximum value of f

Def | if $f(x) \ge f(c)$ for all x in the domain of f, then $f(c)$ is an absolute minimum value of f

thrm | if f is continuous on $[a,b]$ then f has an absolute max/absolute min on $[a,b]$

finding absolute extrema:

1) find critical numbers, c, in $[a,b]$
2) compute $f(c)$ for each c found in (1) and also compute $f(a)$ and $f(b)$
3) the largest function value of (2) is the absolute max and the smallest function value in (2) is the absolute min

EX find absolute extrema of $f(x) = x^2$ on the interval $[-1, 2]$

$f'(x) = 2x \qquad c = x = 0$ critical value

$$\left.\begin{aligned} f(c) &= 0 \\ f(a=-1) &= 1 \\ f(b=2) &= 4 \end{aligned}\right]$$ function values at critical points and endpoints

$f(c) = 0$ absolute minimum on $[-1, 2]$

$f(b) = 4$ absolute maximum on $[-1, 2]$

EX find the absolute max and absolute min of $F(x) = x^3 - 2x^2 - 4x + 4$ on the interval $[0, 3]$

$f'(x) = 3x - 4x - 4 = (3x+2)(x-2)$

$\underbrace{x = -2/3}, \quad x = 2$

outside of interval so ~~we~~ ignore it

$$\left.\begin{aligned} f(2) &= 8 - 2(4) - 4(2) + 4 = -4 \\ f(0) &= 4 \\ f(3) &= 1 \end{aligned}\right]$$ function values at critical points and endpoints

$f(2) = -4$ absolute ~~max~~ min on $[0, 3]$

$f(0) = 4$ absolute max on $[0, 3]$

EX

Profit from manufacturing and selling x units given by

$$P(x) = -.02x^2 + 300x - 200{,}000 \qquad 0 \le x \le 20{,}000$$

how many units should be manufactured and sold in order to maximize profits?

we are looking for the absolute maximum of $P(x)$ on $[0, 20000]$

$$P'(x) = -.04x + 300$$

$x = 7500$ is the critical value

$P(7500) = 925000$

$P(0) = -200{,}000$

$P(20000) = -2200\,000$

absolute maximum profit when 7,500 units are sold and the associated profit is \$925,000

EX velocity of airflow through windpipe

$V(r) = kr^2(R-r)$ with constants K and R

where R represents the maximum radius

In order to execute a cough, the windpipe contracts (shrinks its radius). Find the maximum velocity of a cough

• critical ~~nu~~mbers

$$V'(r) = 2kr(R-r) - kr^2$$

$$= kr(-3r+2R)$$

$r=0$ $\quad r = \frac{2}{3}R$

endpoint

so for our purposes, not a critical value

$V(0) = 0$

$V(R) = 0$

$V(\frac{2}{3}R) = \frac{4K}{27}R^3$

maximum velocity: $\left(\frac{2}{3}R, \frac{4K}{27}R^3\right)$