Section 5.2

def $y = \log_b x$ if and only if $x = b^y$

EX

$\log_{10} 100 = 2$ because $100 = 10^2$

$\log_5 125 = 3$ because $125 = 5^3$

$\log_3 \frac{1}{9} = -2$ because $\frac{1}{9} = 3^{-2}$

$\log_{20} 20 = 1$ because $20 = 20^1$

$\log_x 1 = 0$ because $1 = x^0$

EX solve:

$\log_3 x = 4$ $\quad 3^4 = x = 81$

$\log_{16} 4 = x$ $\quad 16^x = 4 \Rightarrow (4^2)^x = 4 \Rightarrow 4^{2x} = 4^1 \Rightarrow x = \frac{1}{2}$

$\log_x 8 = 3$ $\quad x^3 = 8 \Rightarrow x^3 = 2^3 \Rightarrow x = 2$

Notation

$\log X = \log_{10} X$

$\ln X = \log_e X$

Properties of logs

m, n positive
$b > 0$
$b \neq 1$

- $\log_b m \cdot n = \log_b m + \log_b n$
- $\log_b \frac{m}{n} = \log_b m - \log_b n$
- $\log_b m^n = n \log_b m$
- $\log_b 1 = 0$
- $\log_b b = 1$

Caution

Don't confuse

$\log \frac{m}{n}$ with $\frac{\log m}{\log n}$

↑ rule works

↑ rule doesn't work

Ex

$\log(2\cdot 3) = \log 2 + \log 3$

$\ln \frac{5}{3} = \ln 5 - \ln 3$

$\log \sqrt{7} = \log 7^{1/2} = \frac{1}{2}\log 7$

$\log_5 1 = 0$

$\log_{45} 45 = 1$

Ex given: $\log 2 \approx 0.3010$
$\log 3 \approx 0.4771$
$\log 5 \approx 0.6990$

find:

$\log 15 = \log 3\cdot 5 = \log 3 + \log 5 = 1.1761$

$\log 7.5 = \log\left(\frac{15}{2}\right) = \log 15 - \log 2 = 1.1761 - 0.3010$
$= 0.8751$

$\log 81 = \log 9^2 = \log (3^2)^2 = \log 3^4 = 4\log 3 = 4(0.4771)$
$= 1.9084$

$\log 50 = \log 5\cdot 5.2 = \log 5^2 + \log 2 = 2\log 5 + \log 2$
$= 2(0.6990) + 0.3010$
$= 1.6990$

$= \log 5\cdot 10$
$= \log 5 + \log 10$
$= (0.6990) + 1$
$= 1.6990$

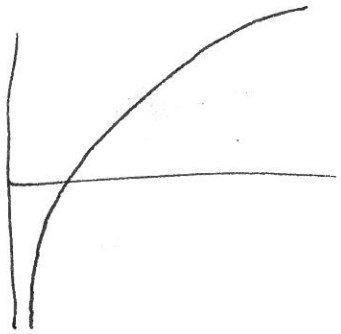
Simplify

$$\log_3 x^2 y^3 = \log_3 x^2 + \log_3 y^3$$
$$= 2\log_3 x + 3\log_3 y$$

$$\log_2 \frac{x^2+1}{2^x} = \log_2(x^2+1) - \log_2 2^x$$
$$= \log_2(x^2+1) - x\log_2 2$$
$$= \log_2(x^2+1) - x$$

$f(x) = \log_b x$

Domain: all positive numbers
range: $(-\infty, \infty)$
through: $(1,0)$



$e^{\ln x} = x$

$\ln e^x = x$

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solve

$2e^{x+2} = 5$

$e^{x+2} = 5/2$

$\log e^{x+2} = \log 5/2$

$x+2 = \log 2.5$

$x = -2 + \log 2.5$

$5\ln x + 3 = 0$

$5\ln x = -3$

$\ln x = -3/5 = -.6$

$e^{\ln x} = e^{-.6}$

$x = e^{-.6} \approx 0.55$