Section 5.4

$f(x) = e^x$

$f'(x) = \frac{d}{dx}[e^x]$

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{e^{x+h} - e^x}{h}$

$= \lim_{h \to 0} \frac{e^x e^h - e^x}{h}$

$= \lim_{h \to 0} \frac{e^x(e^h - 1)}{h}$

$= e^x \underbrace{\lim_{h \to 0} \frac{e^h - 1}{h}}_{\to 1}$

$= e^x 1$

$= e^x$

Ex

$f(x) = x^2 e^x$

$f'(x) = (2x)(e^x) + (e^x)(x^2)$ product rule

$= xe^x(2 + x)$

EX

$g(x) = (e^x + 2)^{3/2}$

$g'(x) = \frac{3}{2}(e^x + 2)^{1/2} \frac{d}{dx}[e^x + 2]$

$= \frac{3}{2}(e^x + 2)^{1/2} e^x$

$\frac{d}{dx}\left[e^{f(x)}\right] = e^{f(x)} f'(x)$

$\frac{d}{dx}\left[e^{ax}\right] = a e^{ax}$

Ex

$$f(x) = e^{2x}$$

$$f'(x) = 2e^{2x}$$

$$g(x) = e^{(2x^2+x)}$$

$$g'(x) = e^{(2x^2+x)} \frac{d}{dx}\left[2x^2 + x\right]$$

$$= e^{(2x^2+x)}(4x+1)$$

Ex

$$f(x) = xe^{-2x}$$

$$f'(x) = \frac{d}{dx}[x]e^{-2x} + \frac{d}{dx}\left[e^{-2x}\right]x$$

$$= e^{-2x} + -2e^{-2x}x$$

$$= e^{-2x}(1-2x)$$

Ex

$$f(x) = \frac{e^x}{e^x + e^{-x}}$$

$$f'(x) = \frac{\frac{d}{dx}[e^x](e^x+e^{-x}) - \frac{d}{dx}\left[e^x+e^{-x}\right]e^x}{(e^x+e^{-x})^2} = \frac{e^x(e^x+e^{-x}) - (e^x-e^{-x})e^x}{(e^x+e^{-x})^2}$$

$$= \frac{\cancel{e^{2x}} + e^0 - \cancel{e^{2x}} + e^0}{(e^x+e^{-x})^2} = \frac{1+1}{(e^x+e^{-x})^2} = \frac{2}{(e^x+e^{-x})^2}$$

find the concavity and inflection points

find regions of inc/dec

$f(x) = e^{-x^2}$

$f'(x) = e^{-x^2} \frac{d}{dx}[-x^2] = e^{-x^2}(-2x) = -2xe^{-x^2}$

$f''(x) = \frac{d}{dx}[-2x]e^{-x^2} + \frac{d}{dx}[e^{-x^2}](-2x)$

$= -2e^{-x^2} + (-2xe^{-x^2})(-2x)$

$= -2e^{-x^2} + 4x^2 e^{-2x^2}$

$= (4x^2 - 2)e^{-2x^2}$

Inc / Dec

$f'(x) = 0 \qquad 0 = -2xe^{-x^2}$

$x = 0$

	TV	+/-	
$(-\infty, 0)$	-1	+	inc
$(0, \infty)$	1	-	dec

$f''(x) = 0 \qquad 0 = (4x^2 - 2)e^{-2x^2} \qquad 4x^2 = 2 \qquad x^2 = 1/2 \quad x = \pm \frac{1}{\sqrt{2}}$

	TV	+/-	
$(-\infty, -1/\sqrt{2})$	-1	+	up
$(-1/\sqrt{2}, 1/\sqrt{2})$	0	-	down
$(1/\sqrt{2}, \infty)$	1	+	up

inflection points $x = -1/\sqrt{2}, 1/\sqrt{2}$