Section 5.6

$Q(t) = Q_0 e^{kt}$ Exponential Growth

* $Q(0) = Q_0 e^0 = Q_0$ initial value
* as t grows, $Q(t)$ grows rapidly and unbounded
* k is the rate of growth

Ex 10,000 bacteria initially present
60,000 bacteria after 2 hours

(a) how many after 4 hours?

(b) what is the growth-rate after 4 hours?

a)
$$60000 = 10000 e^{k(2)}$$
$$6 = e^{2k}$$
$$\ln 6 = \ln e^{2k}$$
$$\ln 6 = 2k$$
$$k = \frac{(\ln 6)}{2} \approx .8959$$
$$Q(t) = 10000 e^{.8959t}$$
$$Q(4) = 10000 e^{.8959(4)} \approx 360{,}000$$

b)
$$Q'(t) = k Q(t) = .8959 e^{.8959t}$$
$$Q'(4) \approx 322{,}500$$

$Q(t) = Q_0 e^{-kt}$ Exponential decay

$Q'(t) = -k\,Q(t)$

Half-life: time it takes half of a sample to go away

if we start at 100% of a sample, the time it takes to get to 50% is the half life

EX/ half-life of radium is 1600 years if the initial starting amount is 200 milligrams, how much is left after 800 years

$Q(t) = Q_0 e^{-kt}$

$.5 = 1\, e^{-k(1600)}$

$\ln .5 = \ln e^{-k1600}$

$\ln .5 = -k1600$

$k = \frac{-\ln .5}{1600} \approx 0.0004332$

$Q(t) = Q_0 e^{-0.0004332t}$

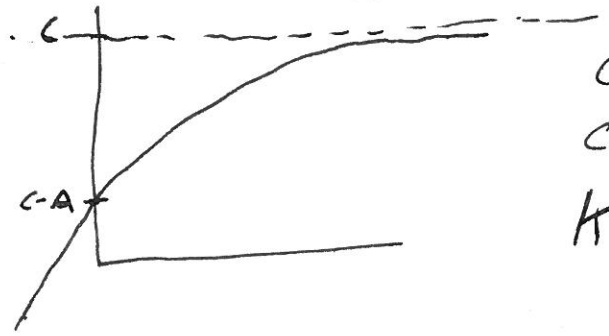
$Q(800) = 200 e^{-0.0004332(800)}$

≈ 141.42

~~Learning~~

Learning Curve

$$Q(t) = C - Ae^{-kt}$$



C : saturation point

C-A : initial knowlage

k : rate of learning

Logistic Growth

$$Q(t) = \frac{A}{1 + Be^{-kt}}$$