Section 6.4

Fundamental theorem of calculus

Let f be continuous on $[a,b]$. Then
$$\int_a^b f(x)\,dx = F(b) - F(a)$$

where $F(x)$ is any anti-derivative (ie $F'(x) = f(x)$)

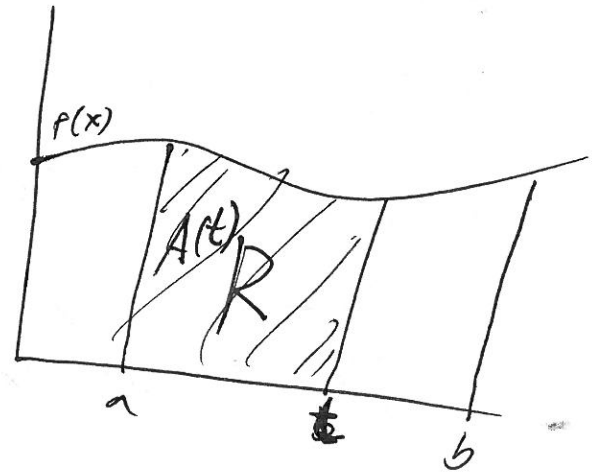
notation:
$$F(x)\Big|_{x=a}^{x=b} = F(b) - F(a)$$

or

$$\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a)$$

Explanation

let $A(t)$ be the area of region R



consider ~~A(t)B~~

$A(t+h)$
which is the area from $x=a$ to $x=t+h$

$A(t+h)-A(t)$ is the area between $t+h$ and t

$$A(t+h)-A(t) \approx h\, f(t)$$

or

$$f(t) \approx \frac{A(t+h)-A(t)}{h}$$

$$\lim_{h\to 0} \frac{A(t+h)-A(t)}{h} = A'(t) = f(t)$$

so A is an anti-derivative of f

$A(x) = F(x) + C$ where F is "the" antiderivative of f

$A(a) = 0$ so $0 = F(a) + C$ or $C = -F(a)$

$$A(b) = F(b) + C = F(b) - F(a)$$

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

P3) consider $f(x) = x$ on the interval $[1, 3]$ use FTC to find R, the area under $f(x)$ on the interval

$$A = \int_1^3 x\,dx = \frac{1}{2}x^2 \Big|_{x=1}^{x=3} = \left(\tfrac{1}{2}(3)^2\right) - \left(\tfrac{1}{2}(1)^2\right)$$

$$= 9/2 - 1/2 = 8/2 = 4$$

EX Find the area of the region R, the area under $f(x) = x^2+1$ between $x=-1$ and $x=2$

$$\int_{-1}^{2} (x^2+1)\,dx = \left(\tfrac{1}{3}x^3 + x\right)\Big|_{x=-1}^{x=2}$$

$$= \left(\tfrac{1}{3}(2)^3 + (2)\right) - \left(\tfrac{1}{3}(-1)^3 + (-1)\right)$$

$$= 8/3 + 2 + 1/3 + 1$$

$$= 9/3 + 3$$

$$= 6$$

EX $$\int_1^2 \left(\frac{1}{x} - \frac{1}{x^2}\right)dx = \int_1^2 \left(\frac{1}{x} - x^{-2}\right)dx$$

$$= \left(\ln|x| + \frac{1}{x}\right)\Big|_{x=1}^{x=2} = \left(\ln(2) + \frac{1}{2}\right) - \left(\underset{\downarrow\, 0}{\ln(1)} + \frac{1}{1}\right)$$

$$= \ln(2) + 1/2 - 1 = \ln(2) - 1/2$$

Ex Clark County (contains las Vegas)

grew at a rate of

$R(t) = 133680t^2 - 178788t + 234633 \quad (0 \leq t \leq 3)$

per decade between 1970 and 2000

what was the net change in population between 1980 and 1990

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if we recall back to when we first discussed integration, we observed that the integral of a rate gives the quantity

so integral of population rate is population total

$$\int_1^2 R(t)\,dt = \frac{133680}{3}t^3 - \frac{178788}{2}t^2 + 234633t \Big|_{t=1}^{t=2}$$

$$= 44560t^3 - 89394t^2 + 234633t \Big|_{t=1}^{t=2}$$

$$= 44560(2)^3 - 89394(2)^2 + 234633(2)$$
$$- 44560(1)^3 + 89394(1)^2 - 234633(1)$$

$$= 278371$$

Net change

the net change of f over $[a, b]$ is

$\int_a^b f'(x)\,dx = f(b) - f(a)$ provided f' is continuous on $[a, b]$