section 6.5

properties of definite integrals

$$\int_a^a f(x)\,dx = 0$$

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

$$\int_a^b c\,f(x)\,dx = c\int_a^b f(x)\,dx$$

$$\int_a^b [f(x) \pm g(x)]\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$$

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx \qquad (a < c < b$$

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integration by subsditution
(u-substitution)

EX

$$\int_{x=0}^{x=4} x\sqrt{9+x^2}\,dx$$

$$U = 9+x^2$$

$$\frac{du}{dx} = 2x$$

$$du = 2x\,dx$$

$$x\,dx = \frac{1}{2}\,du$$

$$\int_9^{25} \sqrt{u}\ \frac{1}{2}\,du = 98/3$$

alternative method

$$\int_0^4 x\sqrt{9+x^2}\,dx$$

$$= \frac{1}{3}(9+x^2)^{3/2}\Big|_0^4$$

$$= 98/3$$

$u = 9+x^2$

$\frac{du}{dx} = 2x$

$\frac{1}{2}du = x\,dx$

$$I = \int x\sqrt{9+x^2}\,dx$$

$$= \int \sqrt{u}\,\frac{1}{2}\,dx$$

$$= \frac{1}{2}\int u^{1/2}\,dx$$

$$= \frac{1}{2}\left(\frac{1}{3/2}u^{3/2}\right) + C$$

$$= \frac{1}{2}\,\frac{2}{3}\,u^{3/2} + C$$

$$= \frac{1}{3}u^{3/2} + C$$

$$= \frac{1}{3}(9+x^2)^{3/2} + C$$

EX1

$$\int_{x=0}^{u=2} x e^{2x^2}\,dx$$

$u = 2x^2$ $\quad \frac{du}{dx} = 4x$ $\quad \frac{1}{4}du = x\,dx$

$$= \int_{u=0}^{u=8} e^u \,\frac{1}{4}\,du$$

$$= \frac{1}{4}\int_0^8 e^u\,du$$

$$= \frac{1}{4}e^u\Big|_0^8$$

$$= \frac{1}{4}(e^8 - e^0)$$

$$= \frac{1}{4}(e^8 - 1)$$

EX

$u = x^3+1$

$\frac{du}{dx} = 3x^2$

$\frac{1}{3}du = x^2\,dx$

$$\int_{x=0}^{x=1} \frac{x^2}{x^3+1}\,dx$$

$$\int_{u=1}^{u=2} \frac{1}{u}\,\frac{1}{3}\,du$$

$$\frac{1}{3}\int_1^2 \frac{1}{u}\,du$$

$$\frac{1}{3}\ln|u|\Big|_1^2 = \frac{1}{3}(\ln 2 - \ln 1) = \frac{1}{3}\ln 2$$

find the area under the curve

$f(x) = e^{1/2 x}$ from $x=-1$ to $x=1$.

$$\int_{x=-1}^{x=1} e^{1/2 x}\, dx \qquad u = \tfrac{1}{2}x \qquad \frac{du}{dx} = \tfrac{1}{2} \qquad 2du = dx$$

$$= \int_{u=-1/2}^{u=1/2} e^{u}\, 2du = 2\int_{-1/2}^{1/2} e^{u}\, du = 2\left(e^{u}\Big|_{-1/2}^{1/2} \right)$$

$$= 2\left(e^{1/2} - e^{-1/2}\right) \approx 2.08$$

Discuss: integral of piecewise

$$f(x) = \begin{cases} \sqrt{x} & 0 \le x \le 1 \\ 1/x & 1 < x \le 2 \end{cases}$$

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average value of a function

$$\frac{y_1 + y_2 + \dots + y_n}{n} = \text{average}$$

divide the region into n subintervals

$$\frac{f(x_1) + f(x_2) + \dots + f(x_n)}{n}$$

$$\frac{b-a}{b-a}\left(f(x_1)\tfrac{1}{n} + f(x_2)\tfrac{1}{n} + \dots + f(x_n)\tfrac{1}{n} \right)$$

$$= \frac{1}{b-a}\left(f(x_1)\frac{b-a}{n} + f(x_2)\frac{b-a}{n} + \dots \right)$$

$$= \frac{1}{b-a}\left(f(x_1)\Delta x + f(x_2)\Delta x + \dots \right)$$

$$\lim_{n\to 0} \frac{1}{b-a}\left[f(x_1)\Delta x + \dots \right]$$

$$= \frac{1}{b-a} \lim_{n\to 0}\left[f(x_1)\Delta x + \dots \right]$$

$$= \frac{1}{b-a}\int_a^b f(x)\, dx$$

the average value of f over $[a,b]$ is $\frac{1}{b-a}\int_a^b f(x)\,dx$

EX find the average value of $f(x)=\sqrt{x}$ over the interval $[0,4]$

$$\frac{1}{4-0}\int_0^4 \sqrt{x}\,dx = \frac{1}{4}\left(\frac{1}{3/2}x^{3/2}\Big|_0^4\right)$$

$$= \frac{1}{4}\,\frac{2}{3}\,x^{3/2}\Big|_0^4$$

$$= \frac{1}{6}\left(4^{3/2}-0^{3/2}\right)$$

$$= 4/3$$