DEPARTMENT OF MATHEMATICS
UNIVERSITY OF KANSAS
MATH 121/141, Fall 2008
FINAL EXAM

Your Name: _____ **Instructor:** _____

Problem	Points	SCORE
1 - 10	100	
11	20	
12	20	
13	20	
14	20	
15	20	
16	20	
17	20	
18	20	
19	20	
20	20	
TOTAL	300	

Part A - Multiple Choice Examination

Each right answer is worth 10 points. Select only **one** answer for each problem.

1. The curve $y = xe^{-2x}$ has a horizontal tangent line when x is

   (a) 1 (b) 0 (c) 2
   (d) 0.5 (e) -1 (f) None of the above

2. $f(x)$ is differentiable over the interval $(0, 3)$ with $f(0) = 0$, $f(2) = 3$, and $f(3) = 3$. Which of the following is NOT necessarily true?

   (a) There is a $0 < c < 3$ with $f(c) = 2.5$.
   (b) $f'(x) = 3$ at some point in $[0, 3]$.
   (c) $f'(x)$ is positive at some point in $(0, 3)$.
   (d) $f'(x) = 0$ at some point in $[0, 3]$.
   (e) All of the above statements must be true.

3. Which integral matches the Riemann sum?

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n} \left(\frac{2n+i}{n}\right)^2$$

   (a) $\int_0^1 x\,dx$ (b) $\int_2^3 (2x+1)^2\,dx$ (c) $\int_0^1 \frac{dx}{x^3}$ (d) $\int_0^1 (2+x)^2\,dx$

4. What is the average value of $f(x)$ over the interval $[2, 4]$ if $f(2) = 2$, $f(4) = 8$, and $f'(x) = x$?

   (a) $\frac{28}{3}$ (b) 5 (c) $\frac{7}{3}$ (d) $\frac{14}{3}$ (e) $\frac{56}{3}$ (f) none of the above.

5. $\lim_{x\to 1} \frac{|x-1|}{x^2-1} =$

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) ∞ (d) $-\infty$ (e) does not exist.

6. $\lim_{h\to 0} \frac{1}{h} \ln\left(\frac{3+h}{3}\right) =$

(a) $\frac{1}{3}$ (b) 1 (c) e^3 (d) $\ln 3$ (e) does not exist.

7. If f, f' and f'' are continuous functions on the interval [a,b], such that $f'(a) > 0$, $f'(b) < 0$, and $f''(x) < 0$ for all x in (a,b), then which of the following must be true:

(a) There exist a number c, in (a,b), such that $f(c) = 0$.
(b) There exist a number c in (a,b) such that $f(c) = \frac{a+b}{2}$.
(c) $f(x)$ has a local minimum in the interval (a,b).
(d) $f(x)$ has a local maximum in the interval (a,b).
(e) None of the above.

8. Use Newton's method to solve the equation $x^2 - e^x = 0$, starting with $x_1 = 1$. The next iteration x_2 is:

(a) 1 (b) 0 (c) $\frac{1}{1-e}$ (d) $\frac{1}{2-e}$ (e) None of the above.

9. (**25 points**) Evaluate

$$\int_1^{\sqrt{3}} \arctan(1/x)dx.$$

10. (**25 points**) A rectangular box with a square base and open top is constructed to have volume of 625 cubic inches. The material used to make the bottom costs 4 cents per square inch and the material used to make the sides costs 2 cents per square inch. Find the dimensions of the box that minimizes the total costs. Justify your answer.

11. (**25 points**) A pile of sand with volume $12,000,000\pi$ cubic feet is collapsing. As it collapses it remains in the shape of a cone so that the height decreases as the radius increases and the total volume of the sand remains constant. If the height is decreasing at a constant rate of $4ft/min$, how fast is the radius increasing at the instant the height is 90 feet and the radius is 200 feet?

12. (**25 points**) For the implicitly defined curve

$$x^y = y^x,$$

write the equation of the tangent line at the point $P(2, 2)$.

13. (**30 points**) Compute the limit

$$\lim_{x \to 1+} \left(\frac{x}{x-1} - \frac{1}{\ln(x)} \right)$$

14. (**30 points**) A water tank in the shape of an inverted cone is buried such that the top of the tank is located 3 meters below the ground. The tank has a radius of 10 meters and is 15 meters deep. If the tank is filled to a depth of 12 meters, how much work, measured in joules, is required to empty the tank by pumping all of the water to ground level? (Assume density of water is $1000\,kg/m^3$ and gravitational acceleration is $9.8\,m/sec^2$.)

15. (**30 points**) Consider the region R in the first quadrant bounded by $y = x^3$ and $y = x$.

    - (15 points) Compute the center of mass of the region.
    - (15 points) Compute the volume of the solid obtained by rotating the region R about the y-axis.

16. (**30 points**) Evaluate the improper integral or otherwise show it is divergent

$$\int_0^\infty x^2 e^{-x} dx,$$