Solution to "Paper" Homework #9

Section 5.2 - Prob 3

$$f(x) = e^x - 2, \quad 0 \le x \le 2, \quad n = 4$$

We divide the closed interval $[0, 2]$ in to four subintervals: $[0, \frac{1}{2}]$; $[\frac{1}{2}, 1]$; $[1, \frac{3}{2}]$; and $[\frac{3}{2}, 2]$. Each of these has an equal length of $\Delta x = \frac{2-0}{4} = \frac{1}{2}$. Since the problem requires us to use the midpoints, we go ahead and find these: $\{\frac{1}{2}, \frac{3}{4}, \frac{5}{4}, \frac{7}{4}\}$. The Riemann sum is given by:

$$R_4 = \Delta x \left(f(\frac{1}{2}) + f(\frac{3}{4}) + f(\frac{5}{4}) + f(\frac{7}{4}) \right) \approx 2.322986\,.$$

If you graph/sketch the function, you will see that on interval $[0, 2]$, the function is negative on $[0, \ln(2)]$ and positive on $[\ln(2), 2]$. So this Riemann sum approximates the area underneath the curve (from $x = \ln(2)$ to $x = 2$) minus the area above the curve (from $x = 0$ to $x = \ln(2)$).

Section 5.2 - Prob 42 By the properties of Definite Integrals (see (5.) on page 351))

$$\int_1^5 f(x)dx = \int_1^4 f(x)dx + \int_4^5 f(x)dx\,.$$

Hence

$$\int_1^4 f(x)dx = \int_1^5 f(x)dx - \int_4^5 f(x)dx = 12 - 3.6 = 8.4\,.$$

Section 5.3 - Prob 28

$$\int_0^2 |2x - 1|dx\,.$$

Recall:

$$\begin{aligned} |2x-1| &= \begin{cases} 2x - 1 & \text{if} \quad 2x - 1 \ge 0 \\ -(2x-1) & \text{if} \quad 2x - 1 < 0\,, \end{cases} \\ &= \begin{cases} 2x - 1 & \text{if} \quad \frac{1}{2} \le x \\ -2x + 1 & \text{if} \quad x < \frac{1}{2}\,. \end{cases} \end{aligned}$$

We have:

$$\begin{aligned} \int_0^2 |2x-1|dx &= \int_0^{\frac{1}{2}} |2x-1|dx + \int_{\frac{1}{2}}^2 |2x-1|dx \\ &= \int_0^{\frac{1}{2}} (-2x+1)dx + \int_{\frac{1}{2}}^2 (2x-1)dx \\ &= \frac{1}{4} + \frac{9}{4} = \frac{5}{2}. \end{aligned}$$

Section 5.4 - Prob 16

$$y = \int_{e^x}^0 \sin^3 t\, dt\,.$$

For convenience, let us rename the function as:

$$H(x) = \int_{e^x}^{0} \sin^3 t \, dt\,;$$

so we are looking for $H'(x)$. Notice that:

$$H(x) = \int_{e^x}^{0} \sin^3 t \, dt = -\int_{0}^{e^x} \sin^3 t \, dt\,.$$

Now, if we let

$$g(x) = -\int_{0}^{x} \sin^3 t \, dt \quad (\text{ which has derivative } g'(x) = -\sin^3 x)\,.$$

then,

$$H(x) = -\int_{0}^{e^x} \sin^3 t \, dt = g(e^x)\,.$$

So,

$$H'(x) = g'(e^x) \cdot e^x = -\sin^3(e^x) \cdot e^x\,.$$

Section 5.5 - Prob 34

$$\int \frac{\sin x}{1+\cos^2 x}\, dx$$

Let

$$u = \cos x \qquad \text{and} \qquad \frac{du}{dx} = -\sin x \Rightarrow du = -\sin x \, dx\,.$$

We have:

$$\begin{aligned}
\int \frac{\sin x}{1+\cos^2 x}\, dx &= \int \frac{1}{1+\cos^2 x} \sin(x)\, dx \\
&= -\int \frac{1}{1+(\underbrace{\cos x}_{u})^2} \underbrace{(-\sin(x))\, dx}_{du} \\
&= -\int \frac{1}{1+(u)^2}\, du \\
&= -\tan^{-1}(u) + C = -\tan^{-1}(\cos x) + C
\end{aligned}$$