THE UNIVERSITY OF KANSAS
Department of Mathematics
Math 115 – Spring 2013

The midterm exam is on
WEDNESDAY, MARCH 13, 5:45pm –7:45pm.
See Bb for the exam room.

The exam will covered the following sections from the textbook.

- Chapter 2, Sections 2.2, 2.3, 2.4, 2.5, 2.6.
- Chapter 3, Sections 3.1, 3.2, 3.3, 3.4, 3.5, 3.6, 3.7.

These problems are intended to be used as part of a review for the midterm exam. There is also an assignment of review problems in WebAssign.They are not a substitute for studying the material of the sections and the homework assignments.

MULTIPLE CHOICE QUESTIONS: CIRCLE THE CORRECT ANSWER **No partial credit.**

1. Let f be the function defined by
$$f(x) = \frac{x}{\sqrt{x+1}}.$$
The domain of f is
$$(A)\ [0,\infty) \quad (B)\ (0,\infty) \quad (C)\ [-1,\infty), \quad (D)\ (-1,\infty)$$
(E) None of the above.

2. Let $f(x) = \dfrac{1}{x}$ and $g(x) = \sqrt{x} + 1$ Then $f(g(4))$ is
$$(A)\ \frac{3}{2} \quad (B)\ \frac{1}{3} \quad (C)\ \frac{2}{3} \quad (D)\ \frac{1}{2}$$
(E) None of the above.

3. Let $f(x) = x^2 - 1$ then $\dfrac{f(0+h) - f(0)}{h}$ is
$$(A)\ h \quad (B)\ \frac{h^2-1}{h} \quad (C)\ -h \quad (D)\ \frac{h-1}{h}$$
(E) None of the above.

4. The slope of the tangent line to the curve $y = 0.324x^2 - 0.127x + 2.1$ at $x = 1$ is

$(A)\ 0.775 \quad (B)\ 0.521 \quad (C)\ 2.297 \quad (D)\ 2.424$

(E) None of the above.

5. For $f(x) = \sqrt{2 + \sqrt{x}}$, evaluate $f\,'(4)$.

(A) $1/64$ (B) $1/16$ (C) $1/4$ (D) $1/2$

(E) None of the above.

6. For $f(x) = (x^2 - 1)^3$, evaluate $f\,'(0)$.

(A) -1 (B) 0 (C) 2 (D) 1

(E) None of the above.

Problems 7, 8, 9, and 10 correspond to the function $f(x) = \dfrac{x-1}{x^2-1}$

7. The $\lim_{x\to 1} f(x)$ is

$(A)\ 1 \quad (B)\ -1 \quad (C)\ \dfrac{1}{2} \quad (D)\ -\dfrac{1}{2}$

(E) None of the above.

8. The $\lim_{x\to 0} f(x)$ is

$(A)\ 1 \quad (B)\ -1 \quad (C)\ 0 \quad (D)\ 2$

(E) None of the above.

9. The $\lim_{x\to \infty} f(x)$ is

$(A)\ 1 \quad (B)\ 0 \quad (C)\ \dfrac{1}{2} \quad (D)\ 2$

(E) None of the above.

10. The $\lim_{x\to -1^+} f(x)$ is

$(A)\ -1 \quad (B)\ 0 \quad (C)\ -2 \quad (D)\ 1$

(E) None of the above.

Use the graph of the function f below to determine the value (if it exists)of the limits in Problems 11, 12, 13, 14, and 15.

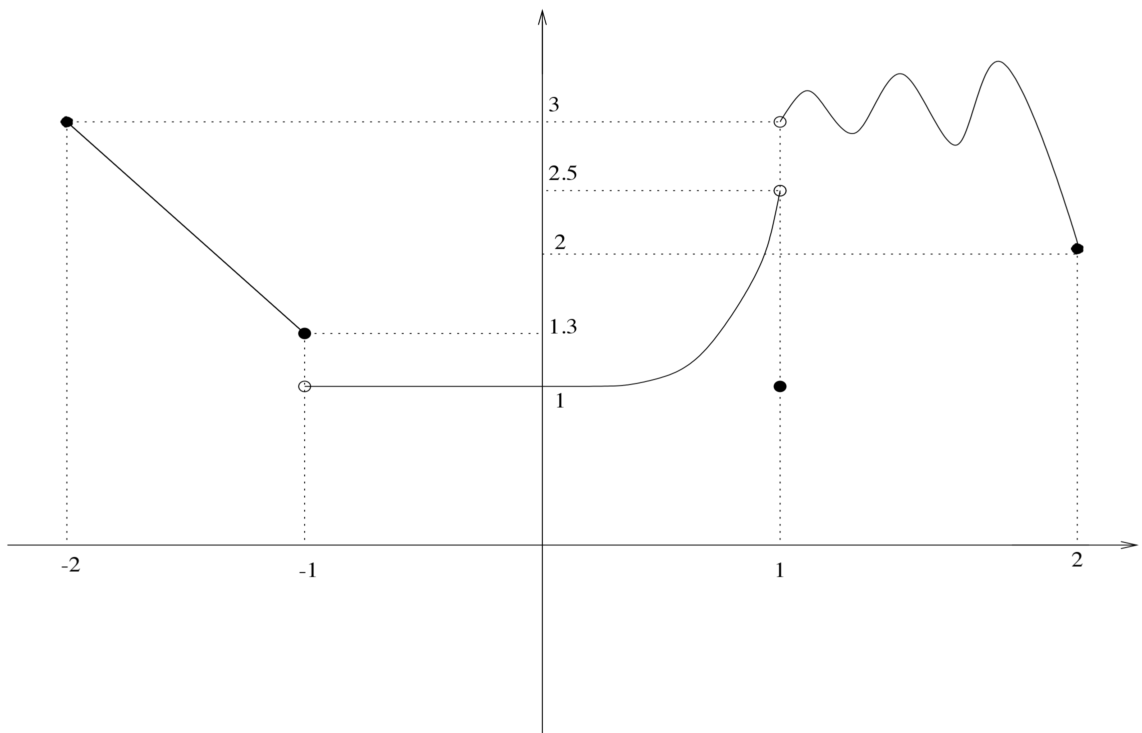

FIGURE 1

11. The $\lim_{x\to -1} f(x)$ is

$(A)\ 2 \quad (B)\ 0 \quad (C)\ 3 \quad (D)\ -1$

(E) None of the above.

12. The $\lim_{x\to 0} f(x)$ is

$(A)\ 0 \quad (B)\ 3 \quad (C)\ 2 \quad (D)$ does not exist.

(E) None of the above.

13. The $\lim_{x\to 2} f(x)$ is

$(A)\ 0 \quad (B)\ 1 \quad (C)\ 2 \quad (D)$ does not exist.

(E) None of the above.

14. The $\lim_{x\to 1^-} f(x)$ is

$(A)\ 0 \quad (B)\ 2 \quad (C)\ -1 \quad (D)$ does not exist.

(E) None of the above.

15. The function $f(x)$ in Figure 1 above is NOT continuous at

$(A)\ x=-2,-1,0,1 \qquad (B)\ x=-2,-1,0,2 \qquad (C)\ x=-2,0,2,3 \qquad (D)\ x=-2,1$

(E) None of the above.

The distance (in feet) covered by a car moving along a straight road t seconds after starting from rest is given by the function $f(t)=2.13t^2+3t$.
Answer Problems 13-14 for this car.

16. The average velocity of the car over the intervals $[23,24]$, $[23,23.1]$, and $[23,23.01]$ are respectively.

$(A)\ \ 241.11, 1481.2, \text{ and } 13901\ ft/sec, \qquad (B)\ 103.11, 10.119, \text{ and } 1.01\ ft/sec$

$(C)\ 103.11, 101.19, \text{ and } 101\ ft/sec, \qquad (D) -100.2, -12.3, \text{ and } -0.32\ ft/sec$

(E) None of the above.

17. The instantaneous velocity of the car when $t=23$ is

$(A)\ 13901 ft/sec \qquad (B)\ 1 ft/sec \qquad (C)\ 100.98 ft/sec \qquad (D)\ 0 ft/sec$

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18. Find the equation of the tangent line to the graph of the function $f(x)=\dfrac{3x+1}{2x+1}$ at $(0,1)$.

$(A)\ y=\frac{1}{4}x-1 \qquad (B)\ y=x+1$

$(C)\ y=\frac{1}{4}x+1 \qquad (D)\ y=-x+1$

(E) None of the above.

The total cost in dollars incurred per week by a company for manufacturing x items is given by the total cost function: $C(x) = 600 + 300x - 0.1x^2 \quad 0 \leq x \leq 300.$ *Answer problems 18 and 19 for the above cost function.*

19. The actual cost incurred for manufacturing the 201st item is

(A) $\$56,860$ $\quad$ (B) $\$259.9$

(C) $\$56,600$ $\quad$ (D) $\$340.1$

(E) None of the above.

20. The marginal cost at $x = 200$ is

$(A)\ 56,860\, dollars/item$ $\quad$ $(B)\ 260\, dollars/item$ $\quad$ $(C)\ 56,600\, dollars/item$ $\quad$ $(D)\ 340\, dollars/item$

(E) None of the above.

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21. The second derivative of $f(x) = (2x-1)^3$ at $x = 1$ is

$(A)\ 24 \quad (B)\ 6 \quad (C)\ 12 \quad (D)\ -24$

22. Given the equation $x^2 - 3xy + y^2 = 0$. Find the derivative $\dfrac{dy}{dx}$ by implicit differentiation.

$(A)\ \dfrac{2x-3y}{2y-3x} \quad (B)\ \dfrac{3y-2x}{2y-3x} \quad (C)\ \dfrac{2x-3}{3y} \quad (D)\ \dfrac{2x-2y}{3x}$

(E) None of the above.

A 10 ft ladder leaning against a wall begins to slide. The bottom of the ladder slides away from the wall at a rate of 1 ft/s when the bottom of the ladder is 6 ft from the wall. See FIGURE 2
Answer Problems 22-23 for this ladder.

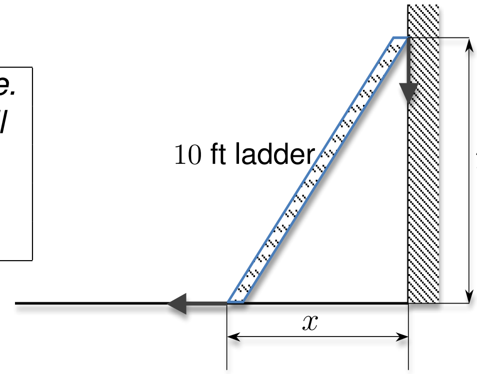

FIGURE 2

23. In this problem, the relationship between x and y is given by the Pythagorean Theorem:

$$x^2 + y^2 = 100.$$

Then the expression for the derivative of y with respect to t, $\frac{dy}{dt}$ is:

$(A)\ \dfrac{dy}{dt} = \dfrac{x}{y}\dfrac{dx}{dt}$ $\quad (B)\ \dfrac{dy}{dt} = -\dfrac{x}{y}\dfrac{dx}{dt}$

$(C)\ \dfrac{dy}{dt} = \dfrac{2x}{y}\dfrac{dx}{dt}$ $\quad (D)\ \dfrac{dy}{dt} = -4\dfrac{x}{y}\dfrac{dx}{dt}$

(E) None of the above.

24. The top of the ladder is sliding down at this instant at a rate of (that is $\frac{dy}{dt}$ when $x = 6$ and $\frac{dx}{dt} = 1$ ft/sec):

$(A)\ \dfrac{dy}{dt} = \dfrac{3}{4}$ ft/sec $\quad (B)\ \dfrac{dy}{dt} = -\dfrac{3}{4}$ ft/sec

$(C)\ \dfrac{dy}{dt} = \dfrac{3}{2}$ ft/sec $\quad (D)\ \dfrac{dy}{dt} = -\dfrac{3}{2}$ ft/sec

(E) None of the above.

The quantity demanded weekly of the Super Titan radial tires is related to its unit price by the equation $p + x^2 = 144$, where p is measured in dollars and x is measured in units of thousand.
Answer Problems 24 – 25 for this price – demand equation.

25. The quantity demanded weekly is changing at a rate of:

$$(A)\ \frac{dx}{dt} = \frac{72}{x}\frac{dp}{dt} \qquad (B)\ \frac{dx}{dt} = -\frac{144}{x}\frac{dp}{dt}$$

$$(C)\ \frac{dx}{dt} = -\frac{1}{2x}\frac{dp}{dt} \qquad (D)\ \frac{dx}{dt} = -\frac{x}{p}\frac{dp}{dt}$$

(E) None of the above.

26. In particular, when $x = 9,\ \ p = 63$, and the price of tire is increasing at the rate of $\$2$ per week, the quantity demanded weekly is changing at a rate of:

$(A)\ \frac{dx}{dt} = 111$ tires per week $\qquad (B)\ \frac{dx}{dt} = -111$ tires per week

$(C)\ \frac{dx}{dt} = 72$ tires per week $\qquad (D)\ \frac{dx}{dt} = -72$ tires per week

(E) None of the above.

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27. Let f be the function defined by $3x^4 - x$. The differential of f is

(A) dx (B) $12x^3 - 1$ (C) $3dx$ (D) $(12x^3 - 1)dx$

(E) None of the above.