MATH 121, Calculus I — Exam I (Spring 2014)

Name: KEY

KU ID No.: ____________________

This exam has a total value of 100 points. There are 9 problems in total to be solved. The first seven are worth 10 points, the remaining two are worth 15 points. This is strictly a closed-book exam. **Be sure to show all work.** If you need to find a derivative, use the **limit definition of the derivative** unless otherwise directed.

Score

# 1	# 2	# 3	# 4	# 5	# 6	# 7	# 8	# 9	Total

1. **[10 points]** Find the exact value of $\lim_{x \to 0} \dfrac{\sqrt{5-x}-\sqrt{5}}{x}$.

$$\lim_{x \to 0} \frac{(\sqrt{5-x}-\sqrt{5})}{x} \frac{(\sqrt{5-x}+\sqrt{5})}{(\sqrt{5-x}+\sqrt{5})} = \lim_{x \to 0} \frac{(\sqrt{5-x}^2-\sqrt{5}^2)}{x(\sqrt{5-x}+\sqrt{5})}$$

$$= \lim_{x \to 0} \frac{\cancel{5}-x-\cancel{5}}{x(\sqrt{5-x}+\sqrt{5})} = \lim_{x \to 0} \frac{-\cancel{x}}{\cancel{x}(\sqrt{5-x}+\sqrt{5})} = \lim_{x \to 0} \frac{-1}{\sqrt{5-x}+\sqrt{5}}$$

$$= {}^{-1}\!/_{2\sqrt{5}}$$

Answer: $\boxed{{}^{-1}\!/_{2\sqrt{5}}}$

2. **[10 points]** Which of the following statements are true? (Since there may be more than one correct answer, determine all correct answers.)

(A) If $\lim_{x \to a} \dfrac{f(x)-f(a)}{x-a}$ exists, then f is differentiable at a.

~~(B)~~ If f is continuous at a, then f is differentiable at a.

~~(C)~~ If $\lim_{x \to a} f(x)$ exists, then f is differentiable at a.

(D) If f is differentiable at a, then $\lim_{x \to a} f(x) = f(a)$.

Answer: $\boxed{A, D}$

3. **[10 points]** Evaluate $\lim_{x\to 0} x^2 \cos(x)$.

$(-1 \le \cos(x) \le 1)\ x^2$

$\Downarrow$

$-x^2 \le x^2\cos(x) \le x^2$

$\lim_{x\to 0} -x^2 \le \lim_{x\to 0} x^2\cos(x) \le \lim_{x\to 0} x^2$

$0 \le \lim_{x\to 0} x^2\cos(x) \le 0$.

$\Rightarrow \lim_{x\to 0} x^2\cos(x) = 0$

Answer: $\boxed{0}$

4. **[10 points]** For what value of the constants a and b is the function f continuous on $(-\infty, \infty)$?

$$f(x) = \begin{cases} \frac{x^2-4}{x-2} & \text{if } x < 2 \\ ax^2 - bx + 3 & \text{if } 2 < x < 3 \\ 2x - a + b & \text{if } x \ge 3 \end{cases}$$

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2} \frac{x^2-4}{x-2} = \lim_{x\to 2} \frac{\cancel{(x-2)}(x+2)}{\cancel{x-2}} = 4$

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2} ax^2 - bx + 3 = 4a - 2b + 3$

$\lim_{x\to 3^-} f(x) = \lim_{x\to 3} ax^2 - bx + 3 = 9a - 3b + 3$

$\lim_{x\to 3^+} f(x) = \lim_{x\to 3} 2x - a + b = 6 - a + b$

Answer: $\boxed{}$

$4 = 4a - 2b + 3$

$1 = 4a - 2b \longrightarrow \times 2$

$9a - 3b + 3 = 6 - a + b$

$9a - 3b = 3 - a + b$

$10a - 4b = 3$

$-8a + 4b = -2$

$+\ 10a - 4b = 3$

$2a + 0 = 1$

$a = 1/2$

$1 = 4(1/2) - 2b$

$1 = 2 - 2b$

$-1 = -2b$

$b = 1/2$

5. [**10 points**] For what values of x does the graph of $f(x) = x^2 - 2$ have a horizontal tangent? You may use derivative rules from chapter 3 if applicable.

$f'(x) = 2x$

$f'(x) = 0$

$2x = 0$

$x = 0$

Answer: $\boxed{x=0}$

6. [**10 points**] Find an equation of the tangent line to the curve $y = 1/x$ at the point $(1,1)$.

$$m = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 0} \frac{\frac{1}{1+h} - \frac{1}{1}}{h} = \lim_{h\to 0} \frac{\frac{1}{1+h} - \frac{1+h}{1+h}}{h}$$

$$= \lim_{h\to 0} \left(\frac{1-(1+h)}{1+h}\right)\frac{1}{h} = \lim_{h\to 0} \frac{-\cancel{h}}{1+h} \cdot \frac{1}{\cancel{h}} = \lim_{h\to 0} \frac{-1}{1+h} = -1$$

$y - y_1 = -1(x - x_1)$

$y - 1 = -1(x - 1)$

$y = -x + 1 + 1$

$y = -x + 2$

Answer: $\boxed{y = -x + 2}$

7. [**10 points**] Find the value of $\lim_{x\to\infty} \frac{3x^2 - x + 2}{x^3 + 3x + 1}$.

$$\lim_{x\to\infty} \frac{(3x^2 - x + 2)}{(x^3 + 3x + 1)} \frac{(1/x^3)}{(1/x^3)} = \lim_{x\to\infty} \frac{3x^2/x^3 - x/x^3 + 2/x^3}{x^3/x^3 + 3x/x^3 + 1/x^3}$$

$$= \lim_{x\to\infty} \frac{\overset{\to 0}{3/x} - \overset{\to 0}{1/x^2} - \overset{\to 0}{2/x^3}}{1 + \underset{\to 0}{3/x^2} + \underset{\to 0}{1/x^3}} = \frac{0 - 0 - 0}{1 + 0 + 0} = \frac{0}{1} = 0$$

Answer: $\boxed{0}$

8. [**15 points**] Let $f(t) = 5t - 9t^2$. Use the *limit definition of the derivative* to find $f'(t)$.

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{[5(x+h) - 9(x+h)^2] - [5x - 9x^2]}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{5x} + 5h - \cancel{9x^2} - 18xh - 9h^2 - \cancel{5x} + \cancel{9x^2}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(5 - 18x - 9h)}{\cancel{h}} = \lim_{h\to 0} 5 - 18x - \overset{\to 0}{9h}$$

$$= 5 - 18x$$

$$f'(x) = 5 - 18x$$

or

$$f'(t) = 5 - 18t$$

Answer: $\boxed{5 - 18t}$

9. [**15 points**] The position function of a particle is given by $s(t) = t/2 + 3$, $t \geq 0$.

(a) When does the particle reach a velocity of 5 m/s? Explain the significance of this.

(b) When does the particle have acceleration 0 m/s^2? Explain the significance of this.

$$v(t) = s'(t) = \lim_{h\to 0} \frac{s(t+h) - s(t)}{h} = \lim_{h\to 0} \frac{[(t+h)/2 + 3] - [t/2 + 3]}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{t/2} - \cancel{t/2} + \cancel{3} - \cancel{3} + h/2}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}\ 1/2}{\cancel{h}} = 1/2$$

when does $v(t) = 5$? $1/2 \neq 5$ so the particle never reaches a speed of 5 m/s. in fact, it has a constant velocity, so it will always travel at a rate of 1/2 m/s.

$$a(t) = v'(t) = \frac{d}{dt}[1/2] = 0$$

the particle always has an acceleration of 0 m/s^2

	$(-\infty, -3-\sqrt{3})$	$(-3-\sqrt{3}, -3+\sqrt{3})$	$(-3+\sqrt{3}, 0)$	$(0, \infty)$
TV	-5	-2	-1	1
$f'(x)$	$-$	$+$	$-$	$+$
	dec	inc	dec	inc

dec on

$\boxed{(-\infty, -3-\sqrt{3}) \cup (-3+\sqrt{3}, 0)}$

Bonus. [**5 points**] On what interval(s) is the function $f(x) = (x^3 + 3x^2)\, e^x$ decreasing? You may use derivative rules from chapter 3 if applicable.

$$f(x) = x^3 e^x + 3x^2 e^x$$

$$f'(x) = 3x^2 e^x + x^3 e^x + 6x e^x + 3x^2 e^x$$

$$= e^x (x^3 + 6x^2 + 6x)$$

$f'(x) = 0$?

$$0 = e^x\, x(x^2 + 6x + 6)$$

$$x = 0,\quad x = \frac{-6 \pm \sqrt{6^2 - 4(6)}}{2(1)}$$

$$= -3 \pm \sqrt{3} \approx -4.7, -1.3$$