Practice Key

MATH 121, Calculus I — Exam I (Spring 2014)

Name: ________________________________

KU ID No.: ________________________________

Lab Instructor: ________________________________

This exam has a total value of 100 points. There are 9 problems in total to be solved. The first seven are worth 10 points, the remaining two are worth 15 points. This is strictly a closed-book exam.

Score

# 1	# 2	# 3	# 4	# 5	# 6	# 7	# 8	# 9	Total

1. **[10 points]** Find the exact value of $\lim_{x \to 0} \frac{\sqrt{3+x} - \sqrt{3}}{x}$.

$$\lim_{x \to 0} \frac{(\sqrt{3+x} - \sqrt{3})}{x} \frac{(\sqrt{3+x} + \sqrt{3})}{(\sqrt{3+x} + \sqrt{3})}$$

$$= \lim_{x \to 0} \frac{(\cancel{3}+x) - \cancel{3}}{x(\sqrt{3+x} + \sqrt{3})}$$

$$= \lim_{x \to 0} \frac{\cancel{x}}{\cancel{x}(\sqrt{3+x} + \sqrt{3})} = \frac{1}{2\sqrt{3}}$$

Answer: $\boxed{1/2\sqrt{3}}$

2. **[10 points]** Which of the following statements are true? (Since there may be more than one correct answer, determine all correct answers.)

(A) If $\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$ exists, then f is differentiable at a.

(B) If f is continuous at a, then f is differentiable at a.

(C) If $\lim_{x \to a} f(x)$ exists, then f is differentiable at a.

(D) If f is differentiable at a, then $\lim_{x \to a} f(x) = f(a)$.

Answer: $\boxed{A, D}$

3. **[10 points]** Suppose that the function g satisfies the following inequality

$$2x \leq g(x) \leq x^4 - x^2 + 2$$

for all values of x. Find the value of $\lim_{x\to 1} g(x)$.

squeeze theorem

$\lim_{x\to 1} 2x \leq g(x) \leq \lim_{x\to 1} x^4 - x^2 + 2$

$2 \leq g(x) \leq 2$

$\Rightarrow \lim_{x\to 1} g(x) = 2$

Answer: $\boxed{2}$

4. **[10 points]** For what value of the constant c is the function f continuous on $(-\infty, \infty)$?

$$f(x) = \begin{cases} cx^2 + 2x & \text{if } x < 2 \\ x^3 - cx & \text{if } x \geq 2 \end{cases}$$

we need

$\lim_{x\to 2} cx^2 + 2x = \lim_{x\to 2} x^3 - cx$

$c(2)^2 + 2(2) = (2)^3 - c(2)$

$4c + 4 = 8 - 2c$

$6c = 4$

$c = 2/3$

Answer: $\boxed{2/3}$

5. **[10 points]** For what values of x does the graph of $f(x) = x^3 - 3x + 1$ have a horizontal tangent?

need to find $f'(x)$

$$f'(x) = \lim_{h \to 0} \frac{[(x+h)^3 - 3(x+h) + 1] - [x^3 - 3x + 1]}{h}$$

$$= \lim_{h \to 0} \frac{h^3 + 3h^2x + 3x^2h \cancel{+x^3} \cancel{-3x} - 3h \cancel{+1} \cancel{-x^3} \cancel{+3x} \cancel{-1}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(h^2 + 3hx + 3x^2 - 3)}{\cancel{h}} = 3x^2 - 3$$

Answer: $\boxed{\pm 1}$

horizontal tangent means $f'(x) = 0$

$3x^2 - 3 = 0 \Rightarrow x^2 - 1 = 0 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$

6. **[10 points]** Find an equation of the tangent line to the curve $y = \sqrt{x}$ at the point $(1, 1)$.

$$m = f'(x) = \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{(\sqrt{x+h} + \sqrt{x})}{(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{\cancel{x}+h \cancel{-x}}{h(\sqrt{x+h} + \sqrt{x})} = \lim_{h \to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{x+h} + \sqrt{x})}$$

$$= \frac{1}{2\sqrt{x}}$$

$m @ (1,1) = f'(1) = 1/2\sqrt{1} = 1/2$

Answer: $\boxed{}$

$Y - Y_1 = m(X - X_1)$

$Y - 1 = 1/2(X - 1)$

$Y = 1/2 X - 1/2 + 1$

$= 1/2 X + 1/2$

7. **[10 points]** Find the value of $\lim_{x\to\infty} \frac{x+2}{\sqrt{9x^2+1}}$.

$$\lim_{x\to\infty} \frac{x+2}{\sqrt{9x^2+1}} \frac{1/x}{1/x} = \lim_{x\to\infty} \frac{x/x + 2/x}{1/x\sqrt{9x^2+1}} = \lim_{x\to\infty} \frac{1+2/x}{\sqrt{9x^2/x^2 + 1/x^2}}$$

$$= \lim_{x\to\infty} \frac{1+2/x^{\to 0}}{\sqrt{9+\underbrace{1/x^2}_{\to 0}}} = \frac{1+0}{\sqrt{9+0}} = 1/3$$

Answer: $\boxed{1/3}$

8. **[15 points]** Let $f(t) = 5t - 9t^2$. Use the *limit definition of the derivative* to find $f'(t)$.

Caution: Do <u>not</u> use the Power Rule to solve this problem.

$$f'(t) = \lim_{h\to 0} \frac{f(t+h)-f(t)}{h} = \lim_{h\to 0} \frac{[5(t+h)-9(t+h)^2]-[5t-9t^2]}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{5t}+5h\cancel{-9t^2}-18th-9h^2\cancel{-5t}\cancel{+9t^2}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(5-18t-9h)}{\cancel{h}} = \lim_{h\to 0} 5-18t-9h^{\to 0}$$

$$= 5-18t$$

Answer: $\boxed{}$

9. [**15 points**] The position function of a particle is given by $s(t) = t^2 - 4.5t$, $t \geq 0$.

(a) When does the particle reach a velocity of 5 m/s?

(b) When does the particle have acceleration 0 m/s^2? Explain the significance of this value of t.

a)

$v(t) = s'(t) = 2t - 4.5$

$2t - 4.5 = 5$

$2t = 9.5$

$t = 4.75$

b)

$a(t) = v'(t) = 2$

the particle always has an acceleration of 2

Bonus. [**5 points**] On what interval(s) is the function $f(x) = x^3e^x$ increasing?

$f'(x) = 3x^2e^x + x^3e^x = e^x(3x^2 + x^3)$

$0 = e^x(3x^2 + x^3)$

$0 = e^x$ can't happen

$0 = 3x^2 + x^3 = x^2(3 + x)$

$x = 0 \quad x = -3$

	$x < -3$	$-3 < x < 0$	$0 < x$
$f'(x)$	$-$	$+$	$+$
	dec	inc	inc