MATH 121, Calculus I — Exam III (Spring 2014)

Name: KEY

KU ID No.: ______________________________

This exam has a total value of 100 points. There are 8 problems in total to be solved. Four of the problems are worth 10 points each, the remaining four problems are worth 15 points each. This is strictly a closed-book exam. If necessary, you may use a calculator. **Be sure to show all work.**

Score

# 1	# 2	# 3	# 4	# 5	# 6	# 7	# 8	Total

1. **[10 points]** If V is the volume of a cube with edge length x and the cube expands as time passes, find dV/dt in terms of dx/dt.

$$V = x^3$$

$$\frac{d}{dt}[V] = \frac{d}{dt}[x^3]$$

$$\frac{dV}{dt} = 3x^2 \frac{dx}{dt}$$

Answer: $\boxed{\frac{dV}{dt} = 3x^2 \frac{dx}{dt}}$

2. **[15 points]** A particle moves along the curve $y = \sqrt{1+x^3}$. As it reaches the point $(2,3)$, the y-coordinate is increasing at a rate of 4 cm/s. How fast is the x-coordinate of the point changing at that instant?

$$\frac{d}{dt}[Y] = \frac{d}{dt}\left[(1+x^3)^{1/2}\right]$$

$$\frac{dY}{dt} = \frac{1}{2}(1+x^3)^{-1/2}(3x^2)\frac{dx}{dt}$$

$$4 = \frac{3(2)^2}{2\sqrt{1+2^3}} \frac{dx}{dt} \rightarrow \frac{dx}{dt} = 2$$

Answer: $\boxed{\frac{dx}{dt} = 2}$

3. [**15 points**]Find the local and absolute maximum and minimum values of $f(x) = 12 + 4x - x^2$ on the interval $[0, 5]$

$f'(x) = 4 - 2x$

$0 = 4 - 2x$

$2x = 4$

$x = 2$

candidates for abs max/min

$f(0) = 12 + 4(0) - (0)^2 = 12$

$f(2) = 12 + 4(2) - (2)^2 = 16$ abs max

$f(5) = 12 + 4(5) - 5^2 = 7$ abs min

$f''(x) = -2$

concave down

$\Rightarrow$ $x = 2$ is local max

Answer: local + absolute max of 16 at $x = 2$

absolute min of 7 at $x = 5$

4. [**10 points**]Suppose that f'' is continuous on $(-\infty, \infty)$.

(a) If $f'(2) = 0$ and $f''(2) = -5$, what can you say about f?

(b) If $f'(6) = 0$ and $f''(6) = 0$, what can you say about f?

a) $f'(2) = 0$ $\Rightarrow$ critical value

$f''(2) = -5$ $\Rightarrow$ concave down

$\Rightarrow$ local max at $x = 2$

b) $f'(6) = 0$ $\Rightarrow$ critical value

$f''(6) = 0$ $\Rightarrow$ inflection point

$\Rightarrow$ inconclusive

Answer:

5. **[10 points]** Evaluate the limit $\lim_{t\to 0} \frac{e^3 t - 1}{t}$

$$\cancel{\lim_{t\to 0} \frac{te^3}{t} - \lim_{t\to 0}\frac{1}{t}}$$

$$\cancel{= e^3 \qquad e^3 \qquad 0}$$

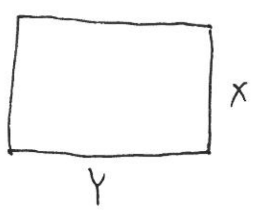
$$= \lim_{t\to 0} \frac{te^3}{t} - \lim_{t\to 0} \frac{1}{t}$$

$$e^t - \left(\lim_{t\to 0^+} \frac{1}{t} \text{ or } \lim_{t\to 0^-} \frac{1}{t} \right)$$

$$= \pm\infty \quad \text{dne}$$

Answer: dne

6. **[15 points]** Find the dimensions of a rectangle with perimeter 100m whose area is as large as possible.



$P = 2X + 2Y = 100 \rightarrow 2Y = 100 - 2X$

$Y = 50 - X$

$A = XY = X(50 - X)$

$A = 50x - x^2$

$A' = 50 - 2x$

$0 = 50 - 2x$

$2x = 50$

$x = 25$

$A'' = -2$

concave down

$\Rightarrow x = 25$ max

if $x = 25$,

$Y = 50 - 25$

$= 25$

Answer: $X = 25$, $Y = 25$

7. **[10 points]**Use Newton's method with $x_1 = -1$ to find x_2, the second approximation of the root to the equation $x^3 + x + 3 = 0$.

trying to solve $f(X) = X^3 + X + 3$ or $f(X) = 0$

$f'(x) = 3x^2 + 1$

$$X_{n+1} = X_n - \frac{f(X_n)}{f'(X_n)} = X_n - \frac{X_n^3 + X_n + 3}{3X_n^2 + 1}$$

$$X_2 = X_1 - \frac{X_1^3 + X_1 + 3}{3X_1^2 + 1} = (-1) - \frac{(-1)^3 + (-1) + 3}{3(-1)^2 + 1} = -5/4$$

Answer: $\boxed{X_2 = -5/4}$

8. **[15 points]**Find the antiderivative $F(x)$ of $f(x) = (x + 1)(2x - 1)$.

$$f(x) = (x+1)(2x-1) = 2x^2 - x + 2x - 1 = 2x^2 + x - 1$$

$$F(x) = \int 2x^2 + x - 1 \, dx = \frac{2}{3}x^3 + \frac{1}{2}x^2 - x + c$$

Answer: $\boxed{\frac{2}{3}x^3 + \frac{1}{2}x^2 - x + C}$